SCOTT WESLEY, Dalhousie Unviersity
Towards an Algebraic and Geometric Theory of Quantum Circuits

The circuit diagrams studied in computer science enjoy a rich mathematical theory. Given a finite set of primitive operators (known as "gates"), a circuit diagram is any operator obtained by composing finitely many gates in sequence or in parallel. Formally, circuits correspond to string diagrams in finitely-generated monoidal categories. A special class of circuit diagrams are the classical reversible circuits, in which gates are invertible matrices over $\mathbb{Z}_2$. It was shown by Toffoli in 1980 that every classical reversible circuit is constructible from a single primitive known as the Toffoli gate. More generally, one can study monoidal groupoids, which characterize all reversible models of computation. For example, the reversible quantum circuits studied by Feynman correspond to the monoidal groupoid of unitary matrices. Since unitary matrices are uncountable, there does not exist an exact universal gate set for quantum computation. However, given both the Toffoli and Hadamard gate, all unitary operators can be simulated.

This talk begins with an introduction to combinatorial circuits as symmetric monoidal string diagrams. The case of classical reversible circuits is recalled. It is then shown how quantum mechanics gives rise to a groupoid of reversible circuits subsuming the classical case. The homsets in this category form groups of circuits with identical wire counts. Presentations for these groups can answer many questions in quantum computing. As a specific example, the 3-qubit dyadic Toffoli+H circuits are considered, whose presentation emerges from the E8 lattice. This presentation, in turn, yields information about the entire sub-groupoid of dyadic Toffoli+H circuits.