Number Theory by early career researchers

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SEDANUR ALBAYRAK, University of Calgary

Quantitative estimates for the size of an intersection of sparse automatic sets

In this talk, I will talk about use of automata theory in answering problems in number theory. In 1844, Catalan conjectured that the set consisting of natural numbers of the form $2^n + 1$, $n \geq 0$ and the set consisting of powers of 3 has finite intersection. In fact, we can answer such question in more generality, that is, instead of 2 and 3, we can show this for k and ℓ that are multiplicatively independent (meaning if $k^a = \ell^b$, then $a = b = 0$). In automata-theoretic terms, these sets described above are sparse 2-automatic and sparse 3-automatic sets, respectively. In fact, a sparse k-automatic set can be more complicated than having elements that are of the form k^n or $k^n + 1$, and hence, we are answering a more general question. Moreover, we also prove our result in a multidimensional setting in line with the existing results in the theory of formal languages and finite automata. We show that the intersection of a sparse k-automatic subset of $\mathbb{N}^d$ and a sparse ℓ-automatic subset of $\mathbb{N}^d$ is finite and we give effectively computable upper bounds on the size of the intersection in terms of data from the automata that accept these sets.

FÉLIX BARIL.BOUDREAU, University of Lethbridge

Value-Distribution of Logarithmic Derivatives of Real Quadratic Dirichlet L-functions over the Projective Line

Let $\mathbb{F}_q(t)$ be a rational function field over a finite field $\mathbb{F}_q$. To each monic irreducible polynomial D in $\mathbb{F}_q[t]$, we can attach a Kronecker symbol χ_D and this is a real quadratic Dirichlet character. We can then define the associated Dirichlet L-function $L(s, \chi_D)$ as some infinite Euler product which, thanks to the work of André Weil, is a polynomial with integer coefficients in the variable $T = q^{-s}$. In her 2019 thesis, Allysa Lumley studied distributions of values of these L-functions for $\mathrm{Re}(s) > \frac{1}{2}$, uncovering that they coincide with some probabilistic random models. Inspired by the seminal work of Yasutaka Ihara on Euler-Kronecker constants of global fields, we study analogous distributions for their logarithmic derivatives $L'(s, \chi_D)/L(s, \chi_D)$. We currently prove that the distribution of these quotients at $s = 1$ is well-approximated by some random model. Moreover, we show that this random model has exponential decay, implying that the distribution function associated with our random model admits a smooth density function. This is ongoing joint work with Amir Akbary (University of Lethbridge).

KÜBRA BENLI, University of Lethbridge

Discrete moments of the derivatives of the Riemann zeta function

In this talk, we will discuss an estimate for a discrete mean value of the Riemann zeta function and its derivatives multiplied by Dirichlet polynomials. Assuming the Riemann Hypothesis, we derive a lower bound for the $2k^{\text{th}}$ discrete moment of the derivatives of the Riemann zeta function evaluated at its nontrivial zeros. This talk is based on a joint work with Ertan Elma and Nathan Ng.

ABHISHEK BHARADWAJ, Queen's University

On primitivity and vanishing of Dirichlet series

For a rational valued periodic function, we associate a Dirichlet series and provide a new necessary and sufficient condition for the vanishing of this Dirichlet series specialized at positive integers. This question was initiated by Chowla, and carried out by Okada for a particular infinite sum. Our approach relies on the decomposition of the Dirichlet characters in terms of primitive characters. Using our approach, we find some new family of natural numbers for which a conjecture of Erdős holds and provide some other applications.

MIHIR DEO, University of Ottawa
Signed p-adic L-functions of Bianchi modular forms

Let $p \geq 3$ be a prime number. Let K be an imaginary quadratic field in which p splits. Let $\mathcal{F}$ be a cuspidal Bianchi eigenform of weight (k, k) over K, where $k \geq 2$ is an integer. In this talk, we will discuss two scenarios of the decomposition of unbounded p-adic L-functions into a linear combination of signed p-adic L-functions in the spirit of Lei-Loeffler-Zerbes, Pollack, and Sprung.

The first half of the talk is about decomposing the two-variable p-adic L-functions $L_p(\mathcal{F})$ constructed by Williams for small slope cuspidal Bianchi eigenforms $\mathcal{F}$, which are non-ordinary at both the primes above p.

In the other half, we discuss a work in progress on p-adic Asai L-functions of Bianchi modular forms. We generalize the construction of Loeffler-Williams in the ordinary case to the non-ordinary case, giving rise to unbounded distributions, which we decompose into bounded measures.

ERTAN ELMA, University of Lethbridge
Number of Prime Factors with a Given Multiplicity

For natural numbers $k, n \geqslant 1$, let $\omega_k(n)$ be the number of prime factors of n with multiplicity k. The functions $\omega_k(n)$ with $k \geqslant 1$ are refined versions of the well-known function $\omega(n)$ counting the number of distinct prime factors of n without any conditions on the multiplicities.

In this talk, we will cover several elementary, analytic and probabilistic results about the functions $\omega_k(n)$ with $k \geqslant 1$ and their function field analogues in polynomial rings with coefficients from a finite field. In particular, we will see that the function $\omega_1(n)$ and its function field analogue satisfy the Erdős-Kac Theorem. The results we will see in this talk are based on joint works with Yu-Ru Liu, with Sourabhashis Das, Wentang Kuo and Yu-Ru Liu, and with Greg Martin.

MATHILDE GERBELLI-GAUTHIER, McGill University
An average Sato-Tate for non-tempered representations.

The (now proved) Sato-Tate conjecture predicts the distribution of Hecke eigenvalues of certain non-CM modular forms. After introducing a representation-theoretic reformulation and generalizations to groups beyond GL_2, I'll discuss joint results with Rahul Dalal giving a Sato-Tate result on average for certain non-tempered representations on unitary groups.

SAMPRIT GHOSH, University of Calgary
Minimal Subfields of Elliptic curves

Let E be an elliptic curve defined over a number field K and let L/K be a finite Galois extension with Galois group $G = Gal(L/K)$. Akbary and Murty introduced the idea of a minimal subfield : $K \subseteq M \subseteq L$, minimal, such that $rank\ E(M) = rank\ E(L)$. They gave a description of the possibilities for $Gal(M/K)$ when the rank $E(L)$ is small. In this talk, we'll present results extending this idea and investigate the possibilities for $Gal(M/K)$ when the $rank\ E(L)$ increases from that of $E(K)$ by a small amount. If time permits we'll also venture in the analytic side of things and present some results in connection to the BSD conjecture.

OUSSAMA HAMZA, Western University
On extensions of number fields with given quadratic algebras and cohomology

At the beginning of the century, Labute and Minac introduced a criterion, on presentations of pro-p groups, ensuring that the cohomological dimension is two. Groups with presentations satisfying this condition are called mild.

In this talk, we introduce a new criterion on the presentation of finitely presented pro-p groups which allows us to compute their cohomology groups and infer quotients of mild groups of cohomological dimension strictly larger than two.

We interpret these groups as Galois groups over p-rational fields with prescribed ramification and splitting.

ERIK HOLMES, University of Toronto
Shapes and asymptotics in number theory

In this talk we investigate a possible connection between the distribution of shapes of number fields and Malle's conjecture. Specifically we will discuss joint work with Rob Harron in which we study the shape of non-Galois sextic fields: i.e. the family of sextic fields which have Galois group $C_3 \wr C_2$ and which were the first counter example to Malle's conjecture. We describe our distribution results within this family and show how they relate to the asymptotics of these fields.

ERMAN ISIK, University of Ottawa
Modular approach to Diophantine equation $x^p + y^p = z^3$ over some number fields

Solving Diophantine equations, in particular, Fermat-type equations is one of the oldest and most widely studied topics in mathematics. After Wiles' proof of Fermat's Last Theorem using his celebrated modularity theorem, several mathematicians have attempted to extend this approach to various Diophantine equations and number fields over several number fields.

The method used in the proof of this theorem is now called the "modular approach", which makes use of the relation between modular forms and elliptic curves. I will first briefly mention the main steps of the modular approach, and then report our asymptotic result (joint work Ozman and Kara) on the solutions of the Fermat-type equation $x^p + y^p = z^3$ over various number fields.

MARTI ROSET JULIÀ, McGill University
The Gross–Kohnen–Zagier theorem via p-adic uniformization

Let S be a set of rational places of odd cardinality containing infinity and a rational prime p. We can associate to S a Shimura curve X defined over $\mathbb{Q}$. The Gross–Kohnen–Zagier theorem states that certain generating series of Heegner points of X are modular forms of weight $3/2$ valued in the Jacobian of X. We will state this theorem and outline a new approach to prove it using the theory of p-adic uniformization and p-adic families of modular forms of half-integral weight. This is joint work in progress with Lea Beneish, Henri Darmon and Lennart Gehrmann.

GREGORY KNAPP, University of Calgary
Polynomial Root Separation and Mahler Measure

In 1964, Mahler proved a valuable lower bound on the separation of a polynomial—the minimal distance between distinct roots of that polynomial—in terms of the Mahler measure of that same polynomial. Many authors, including Bugeaud, Dujella, Fang, Koiran, Pejkovic, Rump, and Salvy have improved, generalized, or investigated the sharpness of this lower bound. However, little attention has been paid to upper bounds on separation in terms of Mahler measure. In this talk, we examine some data on the distribution of separation against Mahler measure, we make a conjecture about an upper bound on separation in terms of Mahler measure, and we describe our partial results which prove that conjecture in certain cases.

JONATHAN LOVE, McGill University
On isospectral quaternion orders

Schiemann proved in 1997 that a 3-dimensional integral lattice is determined up to isometry by the number of elements of each norm. However, in all higher dimensions, there exist many pairs of non-isometric lattices that are isospectral, meaning they have the same number of elements of norm n for all integers n (equivalently, they have the same theta function). Given a quaternion algebra B_p over $\mathbb{Q}$ ramified at a single finite prime p, we show that if two maximal orders of B_p are isospectral, then they are isomorphic. This is joint work with Eyal Goren.

SOHEIL MEMARIANSORKHABI, University of Toronto
Growth Rate of Rational Points on Non-Compact Complex Ball Quotients

Let X be a complex ball quotient by a nonuniform neat lattice in $PU(n, 1)$. Using hyperbolic geometry, we provide a uniform lower bound on the volume of subvarieties of X in terms of a geometric quantity of X called systole. This has an arithmetic consequence: Suppose that the toroidal compactification of X is defined over a number field K. Then, with a mild assumption on X, the systole of X controls the growth rate of K-rational points on X.

MOHAMMADREZA MOHAJER, University of Ottawa
P-adic periods and p-adic subgroup theorem for 1-motives

We define a countable space of p-adic periods of 1-motives with good reduction using the crystalline-de Rham comparison isomorphism and we state a p-adic period conjecture that is analogous to the classical periods. To define these periods, we need to find a "suitable" Betti-like $\bar{\mathbb{Q}}$-structure inside the crystalline realisation. We show that these periods come from p-adic integration theory that we developed for 1-motives with good reduction from the classical Fontaine-Messing p-adic integration theory. Also, we prove the p-adic subgroup theorem for 1-motive that similar to classical periods it implies that the p-adic period conjecture holds for 1-motives with good reduction.

ISABELLA NEGRINI, University of Toronto
A Shintani map for rigid cocycles

Rigid cocycles were defined in 2017 by Darmon and Vonk and give a promising framework to extend the theory of complex multiplication to real quadratic fields. They share striking parallels with modular forms, and their generalizations are the main ingredient in the emerging p-adic Kudla program. In previous work I showed how to build a map from half-integral weight modular forms to rigid cocycles in the style of the Shimura lift. In this talk I will show how to construct a map going in the opposite direction, in the style of the Shintani lift.

DAVID NGUYEN, Queen's University
Shifted convolutions and applications

Many problems in analytic number theory can be reduced to studying a suitable average of the shifted convolution $a(n)a(n+h)$, where $a(n)$ is the coefficients of interest. Determining which average is sufficient is part of the challenge. In this talk, I will survey my works on the shifted convolution problem when the coefficients $a(n)$ is the 3-fold divisor function and discuss an application.

MISHTY RAY, University of Calgary
Introduction to geometry of local Arthur packets

Arthur packets help describe constituents of square integrable automorphic representations. When Arthur initially established his work, the local meaning was of interest. Adams, Barbasch, and Vogan proposed a geometric characterization of local Arthur packets for real groups, and Vogan's subsequent work established this perspective for p-adic groups. In this talk, we will see this geometric perspective in the language of Cunningham et.al. We will report on current progress and future directions of research in this area.

SUBHAM ROY, Université de Montréal
Areal Mahler measure of multivariable polynomials

The (logarithmic) Mahler measure of a non-zero rational polynomial P in n variables is defined as the mean of $\log |P|$ (with respect to the normalized arclength measure) restricted to the standard n-torus ($\mathbb{T}^n = \{(x_1, \ldots, x_n) \in (\mathbb{C}^*)^n : |x_i| = 1, \forall\, 1 \leq$

$i \leq n\}$). It has been related to special values of L-functions. Pritsker (2008) defined a natural counterpart of the Mahler measure, which is obtained by replacing the normalized arclength measure on the standard n-torus by the normalized area measure on the product of n open unit disks. It inherits many nice properties, such as the multiplicative ones. In this talk, we will investigate some similarities and differences between the two. We will also discuss some evaluations of the areal Mahler measure of multivariable polynomials, which also yields special values of L-functions. This is a joint work with Prof. Matilde Lalìn. If time permits we will also define and explore the Zeta Areal Mahler measure.

CHI HOI (KYLE) YIP, University of British Columbia
Additive decompositions of multiplicative subgroups

A celebrated conjecture of Sárközy asserts that if p is a sufficiently large prime, then the set of non-zero squares in $\mathbb{F}_p$ has no non-trivial additive decomposition, that is, it cannot be written as $A + B = \{a + b : a \in A, b \in B\}$, where $A, B \subset \mathbb{F}_p$ and $|A|, |B| \geq 2$. The conjecture is widely open. In this talk, I will focus on the restricted sumset analog of Sárközy's conjecture. More precisely, we show that if $q > 13$ is an odd prime power, then the set of nonzero squares in $\mathbb{F}_q$ cannot be written as a restricted sumset $A \hat{+} A$. More generally, I will discuss related results for multiplicative subgroups over finite fields.

XIAO ZHONG, University of Waterloo
Preimages Question for Surjective Endomorphisms on $(\mathbb{P}^1)^n$

Let K be a number field and let $f : (\mathbb{P}^1)^n \to (\mathbb{P}^1)^n$ be a dominant endomorphism defined over K. We show that if V is an f-invariant subvariety (that is, $f(V) = V$) then there is a positive integer s_0 such that $(f^{-s-1}(V) \setminus f^{-s}(V))(K) = \emptyset$ for every integer $s \geq s_0$, answering the Preimages Question of Matsuzawa, Meng, Shibata, and Zhang in the case of $(\mathbb{P}^1)^n$.