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Weak solutions to diffusion equation with piecewise constant diffusivity

A wide class of weak solutions to the steady inhomogeneous diffusion equation is constructed in three sets of coordinates: polar, spherical and elliptic. This framework is relevant in applications involving fluid flow in porous media, but is also very interesting mathematically. Their existence is shown to depend on the geometric layout of the domain, i.e. the particular division into sub-domains, as well as the diffusivity assigned to each sub-domain. The existence hinges on the null and column spaces of a set of matrices, intriguingly identical in all three systems of coordinates. A fixed point of a new type – half stable - half unstable node – is identified with the aid of this class of weak solutions. A variety of flow patterns associated with these solutions allows to explain certain modelling difficulties encountered in large scale environmental applications, such as aquifer sparging wells, natural and landfill gas wells, as well as petroleum and hydraulic wells. One of the prominent properties of these weak solutions is that with the relaxation of the constraint of axial symmetry – that is obviously unrealistic for the foregoing applications – the locus of zero normal flux comprises separatrices connecting isolated stagnation points. The area enclosed is shown to be realistic in stark contrast to the result obtained with an axially symmetric solution, where the locus comprises a curve of stagnation points. The new class of solutions augments known exact solutions to Laplaces's equation in settings where it is separable.