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Around Poisson-Bessel potentials of fractional L^1-Hardy-Sobolev spaces

Let $u_\alpha(x,t)$, $\alpha \in (0,2)$ be the solution of the equation

$$\Delta_{x,t} u_\alpha(x,t) + (1-\alpha)t^{-1}\partial_t u_\alpha(x,t) = 0$$

on $\mathbb{R}^{n+1}_+ = \mathbb{R}^n \times (0,\infty)$ subject to $u_\alpha(x,0) = f(x)$ on $\mathbb{R}^n$. As the endpoint of the Poisson-Bessel potential u_α, the potential $u_0(x,t)$ solves the equation

$$\Delta_{x,t}\big(\ln t^{-1})u_0(x,t)\big) + t^{-1}\partial_t\big((\ln t^{-1})u_0(x,t)\big) = 0$$

on $\mathbb{R}^{n+1}_+$ subject to $u_0(x,0) = f(x)$ on $\mathbb{R}^n$. The main goal of this paper is to characterize a nonnegative measure μ on $\mathbb{R}^{n+1}_+$ such that $f(x) \mapsto u_\alpha(x,t)$ induces a bounded embedding from the fractional L^1-Hardy-Sobolev space $H^{\alpha,1}(\mathbb{R}^n)$, $\alpha \in (0,2)$ into the weak Lebesgue space $WL^q_\mu(\mathbb{R}^{n+1}_+)$, $q \in [1,\infty)$ and $f(x) \mapsto u_0(x,t)$ induces a bounded embedding from the Hardy $H^{0,1}(\mathbb{R}^n)$ into the Lebesgue space $L^q_\mu(\mathbb{R}^{n+1}_+)$, $q \in [1,\infty)$.

Based on these trace principles, we propose $(H^{\alpha,1}, L^q)$ model and $(H^{\alpha,1}, \log)$ model for image denoising, which significantly improve the reconstruction from images polluted by Gaussian noise or Poisson noise compared with the famous Rudin-Osher-Fatemi model.