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On some covering problems related to Borsuk's conjecture

Borsuk's number $f(n)$ is the smallest integer such that any set of diameter 1 in n-dimensional Euclidean space can be covered by $f(n)$ sets of a smaller diameter. Exponential upper bound $f(n) \le \left(\sqrt{\frac{3}{2}} + o(1)\right)^n$ was first obtained by Schramm (1988) and later by Bourgain and Lindenstrauss (1989), while a lower bound $f(n) \ge (1.2 + o(1))^{\sqrt{n}}$ was obtained by Kahn and Kalai (1993).

To obtain an upper bound on $f(n)$, Bourgain and Lindenstrauss provided exponential bounds (both upper and lower) for the Grünbaum's problem – the problem of determining the minimal number of open balls of diameter 1 needed to cover a set of diameter 1.

On the other hand, in order to obtain an upper bound on $f(n)$ Schramm provided an exponential upper bound on the illumination number of n-dimensional bodies of constant width. Kalai (2015) asked for a corresponding lower bound, namely if there exists an n-dimensional convex body of constant width with the illumination number exponential in n.

In this talk I will outline the construction that answers Kalai's question in the affirmative and provide a new lower bound in the Grünbaum's problem. The talk is based on a joint work with Andriy Bondarenko and Andriy Prymak.