## Computational and Geometric Spectral Theory
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**MAXIME FORTIER BOURQUE**, Université de Montréal
*Two counterexamples to a conjecture of Colin de Verdière*

In 1986, Yves Colin de Verdière conjectured that the multiplicity of the first nonzero eigenvalue of the Laplacian on a closed connected Riemannian manifold is bounded by its chromatic number minus one. I will describe two hyperbolic surfaces of genus 10 and 17 that disprove this conjecture. The proof that these surfaces have large multiplicity uses the twisted Selberg trace formula to rule out low-dimensional representations of their isometry group from appearing in the first eigenspace. This is joint work with Émile Gruda-Mediavilla, Bram Petri, and Mathieu Pineault.

**JADE BRISSON**, Université de Neuchâtel
*Tubes and Steklov eigenvalues in negatively curved manifolds*

In this talk, we establish a tubular neighborhood theorem for embedded closed totally geodesic hypersurfaces in a negatively curved manifolds of dimension $n \geq 3$ extending Basmajian's result in 1994 in the hyperbolic setting. We then consider the Steklov eigenvalue problem on compact pinched negatively curved manifolds with totally geodesic boundaries. We show that the first nonzero Steklov eigenvalue is bounded below in terms of the total volume and boundary area when the dimension is at least three. In particular, it shows that Stelov eigenvalues can only tend to zero when the total volume and/or boundary area go to infinity. It can be seen as a counterpart of the lower bound for the first nonzero Laplace eigenvalues on closed pinched negatively curved manifolds of dimension at least three as proved by Schoen in 1982. We provide examples showing that the dependency on both volume and boundary area is necessary. This is a joint work with Ara Basmajian, Asma Hassannezhad and Antoine Métras.

**GRAHAM COX**, Memorial University
*Geometry and topology of spectral minimal partitions*

A minimal partition is a decomposition of a manifold into disjoint sets that minimizes a certain energy functional. In the bipartite case minimal partitions are closely related to eigenfunctions of the Laplacian, but in the non-bipartite case they are difficult to classify, even for simple domains like the square or the circle.

I will present new results that say a partition that minimizes energy locally is in fact a global minimum (in the bipartite case) and a minimum within a certain topological class of partitions in the non-bipartite case. I will also explain how to construct energy-decreasing deformations of a non-minimal partition, giving insight into the geometric structure of the true minimum. This is joint work with Gregory Berkolaiko, Yaiza Canzani, Peter Kuchment and Jeremy Marzuola.

**EVANS HARRELL**, Georgia Tech
*Upper and lower bounds for eigenvalue gaps for Schrödinger operators and quantum graphs*

The fundamental spectral gap is a well-studied object in spectral geometry and quantum theory. I will report on some recent bounds on this quantity for Schrödinger operators on intervals and metric graphs (networks), showing how the size of the gap is affected on the one hand by assumptions on the potential energy such as convexity and on the other by the topological structure of the graph. Differences from the situation with domains will be noted. This work is joint in part with Borthwick and Zhu, and in part with with Ahrami, El Allali, and Kennedy.

**DIMA JAKOBSON**, McGill
*Nodal sets and negative eigenvalues in conformal geometry*

I will survey some old and more recent results on conformal invariants that arise from nodal sets of eigenfunctions of the conformal laplacian, discuss applications to Nirenberg type problems as well as some related questions

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**HANNA KIM**, University of Illinois, at Urbana-Champaign
*Upper Bound on the Second Laplacian Eigenvalue on the Real Projective Space*

In this talk, I prove an upper bound on the second non-zero Laplacian eigenvalue on $n$-dimensional real projective space. The sharp result for 2-dimension was shown by Nadirashvili and Penskoi and later by Karpukhin when the metric degenerates to that of the disjoint union of a round projective space and a sphere. That conjecture is open in higher dimensions, but I will prove it up to a constant factor that tends to 1 as the dimension tends to infinity. Also, I will also talk about calculating the degree of a map on odd-dimensional spheres with the reflection symmetry property.

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**ALAIN DIDIER NOUTCHEGUEME**, Université de Montréal
*Shape Optimisation for Steklov transmission eigenvalues on surfaces*

Consider a curve on a closed surface endowed with a Riemannian metric. The Steklov transmission problem is to find continuous functions which are harmonic away from the curve, and such that the jump of the normal derivative across the curve is proportional to the value of the function. Such functions are called Steklov transmission eigenfunctions, and the corresponding proportionality coefficients are called Steklov transmission eigenvalues. We will discuss shape optimisation questions for these eigenvalues, and highlight some similarities and differences compared to the usual Steklov case. The talk is based on a joint work with Mikhail Karpukhin (UCL).

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**JEFF OVALL**, Portland State University
*Concerning the localization of eigenvectors for the magnetic Laplacian operator*

Over the past 15 years, significant progress has been made in the mathematical understanding of the mechanisms driving the spatial localization of eigenvectors for standard Schrödinger-type operators, $-\Delta + V$, with a rich theory developing around the so-called "localization landscape" function. In contrast, relatively little theory has been developed concerning localization phenomena for the magnetic Schrödinger operator, $(i\nabla + \mathbf{A})^2 + V$. We present some of our recent contributions to this topic, focusing on the magnetic Laplacian, $V = 0$, and providing both theoretical and computational results.

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**HANNA POTGIETER**, Simon Fraser University
*Numerical approximation of the first p-Laplace eigenpair for large p values*

We present an alternating direction method of multipliers (ADMM) algorithm for approximating the first eigenpair of the p-Laplace operator with zero Dirichlet boundary conditions. In this talk we will discuss the p $\to \infty$ limit and its connection to the underlying geometry of our domain. Working with large p values presents numerical challenges against which the ADMM algorithm outperforms a Newton based solver, at least in certain cases. We show some preliminary computational results in 1D, planar domains, and surfaces lying in $\mathbb{R}^3$.

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**FRÉDÉRIC ROCHON**, UQAM
*Torsion on some fibered cusp manifolds*

Given a number field F with ring of integers $O_F$, one can associate to any torsion free subgroup of $SL(2, O_F)$ of finite index a complete Riemannian manifold of finite volume with fibered cusp ends. For natural choices of flat vector bundles on such a manifold, we show that analytic torsion is identified with the Reidemeister torsion of the Borel-Serre compactification. This is used to obtain exponential growth of torsion in cohomology for sequences of congruence subgroups. This is an ongoing joint work with Werner Müller.

**DAVID SHER**, DePaul University
*Bessel function zeroes and Polya's conjecture*

I will discuss some recent results giving uniform bounds for zeroes of Bessel functions and their derivatives. These bounds can be used to analyze the spectrum of the Laplacian on domains with radial symmetry, and in particular, to prove Polya's conjecture for Euclidean balls. This is joint work with N. Filonov (St. Petersburg), M. Levitin (Reading), and I. Polterovich (Montreal).

**VUKAŠIN STOJISAVLJEVIĆ**, Université de Montréal
*Nodal topology and persistence barcodes*

Classical Courant's nodal domain theorem, together with Weyl's law, gives an upper bound on the number of nodal domains of a Laplace-Beltrami eigenfunction in terms of the corresponding eigenvalue. In general, bounds of this type can not exist for linear combinations of eigenfunctions. We will show how, by coarsely counting nodal domains, i.e. by discarding small oscillations, we may obtain a similar upper bound for linear combinations as well. Our method combines the theory of persistence modules and barcodes with multiscale polynomial approximation of functions in Sobolev spaces. Using the same method, we may study coarse topology of a zero set of a function, as well as coarse topology of the set of common zeros of a number of different functions. This allows us to prove a coarse version of Bézout's theorem for linear combination of Laplace-Beltrami eigenfunctions. The talk is based on a joint work with L. Buhovsky, J. Payette, I. Polterovich, L. Polterovich and E. Shelukhin.

**CRAIG SUTTON**, Dartmouth College
*Generic properties of eigenfunctions in the presence of torus actions*

Let $G$ be a compact Lie group acting on a closed manifold $M$. Inspired by work of Uhlenbeck (1976), we explore the generic properties of Laplace eigenfunctions associated to $G$-invariant metrics on $M$. We find that, in the case where $\mathbb{T}$ is a torus acting freely on $M$, the Laplace eigenspaces associated to a generic $\mathbb{T}$-invariant metric are irreducible representations of $\mathbb{T}$. This provides a mathematically rigorous instance of the belief in quantum mechanics that, in the presence of symmetry, non-irreducible eigenspaces are "accidental degeneracies." Turning to nodal sets of Laplace eigenfunctions, we find that for a generic $\mathbb{T}$-invariant metric on $M$ the nodal sets are embedded hypersurfaces. Additionally, under suitable conditions, our framework allows us to prove the existence of numerous examples of Riemannian manifolds for which, modulo a subspace of Weyl density zero, every non-constant Laplace eigenfunction has precisely two nodal domains, which is the minimal possible number.

This is joint work with Donato Cianci (GEICO), Chris Judge (Indiana) and Samuel Lin (Oklahoma).

**JOHN TOTH**, McGill University
*Goodness estimates in microlocally allowable regions*

Let $(M, g)$ be a compact, $C^\omega$ Riemannian surface. Let $\{u_h\}$ be a quantum ergodic (QE) sequence of Laplace eigenfunctions. Then, for every locally asymmetric $C^\infty$ curve $H \subset M$ there exist $C_H > 0$ and $h_0 > 0$ such that for $0 < h < h_0$

$$\|u_h\|_{L^2(H)} \geq e^{-C_H/h}.$$

In particular, such curves do not persist as components of eigenfunction nodal sets. This is joint work with Yaiza Canzani (UNC Chapel Hill).

**DANIEL VENN**, Simon Fraser University
*Surface Partial Differential Equation Solvability and Eigenvalues with Symmetric Meshfree Methods*

We present a novel technique and analysis for investigating the solvability of certain linear partial differential equations (PDEs) using underdetermined Fourier extensions or symmetric Hermite radial basis function methods. The technique applies to surface PDEs as well as flat domain problems.

While much recent work has been completed on using meshfree methods for solving a wide range of PDEs, the spectra of operators discretized using radial basis functions (RBFs) suffers from the presence of non-physical eigenvalues (spurious modes). This makes many RBF methods unhelpful for eigenvalue problems. Our technique provides a rigorously justified process for finding eigenvalues based on a result concerning the norm of a Hermite RBF solution in its native space; specifically, only PDEs with solutions in the native space produce RBF solutions with bounded norms as the fill distance approaches zero.

The approach also works with underdetermined Fourier extensions: our own related approach with certain flexibility and stability advantages. Importantly, both the Fourier extension and Hermite RBF methods for eigenvalues can be used for surface eigenvalue problems. Meshfree methods are desirable for surface problems due to the increased difficulties associated with mesh creation and refinement on curved surfaces.

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**DENIS VINOKUROV**, Université de Montréal
*The first eigenvalue of the Laplacian on orientable surfaces*

The famous Yang-Yau inequality provides an upper bound for the first eigenvalue of the Laplacian on an orientable Riemannian surface solely in terms of its genus $\gamma$ and the area. Its proof relies on the existence of holomorhic maps to $\mathbb{CP}^1$ of low degree. Very recently, Ros was able to use certain holomorphic maps to $\mathbb{CP}^2$ in order to give a quantitative improvement of the Yang-Yau inequality for $\gamma = 3$. In the present paper, we generalize Ros' argument to make use of holomorphic maps to $\mathbb{CP}^n$ for any $n > 0$. As an application, we obtain a quantitative improvement of the Yang–Yau inequality for all genera except for $\gamma = 4, 6, 8, 10, 14$. Later, Ros adjusted some parts of the prove that has lead to even better assymptotic estimates.