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Toward a Three-dimensional Counterpart of Cruse's Theorem

Completing partial latin squares is NP-complete. Motivated by Ryser's theorem for latin rectangles, in 1974, Cruse found conditions that ensure a partial symmetric latin square of order m can be embedded in a symmetric latin square of order n. Loosely speaking, this results asserts that an n-coloring of the edges of the complete m-vertex graph K_m can be embedded in a one-factorization of K_n if and only if n is even and the number of edges of each color is at least $m-n/2$. We establish necessary and sufficient conditions under which an edge-coloring of the complete λ-fold m-vertex 3-graph λK_m^3 can be embedded in a one-factorization of λK_n^3. In particular, we prove the first known Ryser type theorem for hypergraphs by showing that if $n \equiv 0 \pmod 3$, any edge-coloring of λK_m^3 where the number of triples of each color is at least $m/2 - n/6$, can be embedded in a one-factorization of λK_n^3. Finally we prove an Evans type result by showing that if $n \equiv 0 \pmod 3$ and $n \geq 3m$, then any q-coloring of the edges of any $F \subseteq \lambda K_m^3$ can be embedded in a one-factorization of λK_n^3 as long as $q \leq \lambda\binom{n-1}{2} - \lambda\binom{m}{3}/\lfloor m/3 \rfloor$. These results can be restated as results on embedding partial symmetric layer-rainbow latin cubes in partial symmetric layer-rainbow latin cubes where all diagonal entries are empty.