The many facets of random matrix theory
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GIORGIO CIPOLLONI, Princeton University
Logarithmically correlated fields in non-Hermitian random matrices

We prove that for matrices with i.i.d. entries the fluctuations of their eigenvalues converge to a 2D log-correlated field. We study the extremal value of this field and demonstrate its logarithmic dependence on the matrix dimension. I will then explain how a 3D log-correlated field naturally emerges from dynamics on non-Hermitian matrices.

JONATHAN HUSSON, University of Michigan
Generalized empirical covariance matrices and large deviations.

In many applications of random matrix theory, such as Principal Component Analysis or the study of random landscapes, the behavior of the largest eigenvalue is of particular importance. In this talk, we will consider a model of generalized empirical covariance matrix and we will state a large deviation principle for its largest eigenvalue. The main tool of the proof is the use of a spherical integral of rank one as a proxy for this largest, eigenvalue. This makes possible to tackle not only Gaussian entries but also so-called "sharp sub-Gaussian" entries such as Rademacher random variables. We then have a universality phenomenon - which is rather surprising in the large deviation regime - as well as an elegant representation for the rate function. This talk is based on a collaboration with Ben McKenna.

VISHESH JAIN, University of Illinois Chicago
Invertibility of random matrices

Motivated by applications to numerical analysis and the study of the limiting spectral distribution of random matrices, the fundamental problem of establishing the invertibility of random square matrices and providing accompanying lower-tail estimates for the smallest singular value has been intensely studied, especially during the past 20 years. In this talk, we will survey some recent developments in this area from the past few years and highlight possible directions for future investigation.

JUSTIN KO, University of Waterloo
Spectral Phase Transitions in Non-Linear Wigner Spiked Models

In this talk, we discuss the asymptotic behavior of the spectrum of a random matrix where a non-linearity is applied entry-wise to a Wigner matrix perturbed by a rank-one spike with independent and identically distributed entries. In this setting, we show that when the signal-to-noise ratio scale as $N^{\frac{1}{2}(1-1/k_\star)}$, where $k_\star$ is the first non-zero generalized information coefficient of the function, the non-linear spike model effectively behaves as an equivalent spiked Wigner matrix, where the former spike before the non-linearity is now raised to a power $k_\star$. This allows us to study the phase transition of the leading eigenvalues, generalizing part of the work of Baik, Ben Arous and Péché to these non-linear models. We also will explain an application of this result to estimate a low-rank matrix from non-linear and possibly noisy observations. This talk is based on recent and upcoming work with Alice Guionnet, Florent Krzakala, Pierre Mergny, and Lenka Zdeborová.

BENJAMIN LANDON, University of Toronto
Regularity conditions in the CLT for random matrices

A classical result in random matrix theory is that for wide classes of Hermitian matrices, Linear Spectral Statistics of the form $\sum_{i=1}^{N} f(\lambda_i)$ have asymptotic Gaussian fluctuations in the limit of large dimension $N \to \infty$. For Wigner matrices, the limiting variance is a Sobolev-type norm, the $H^{1/2}$ norm of the function expressed in the basis of Chebyshev polynomials.

Conjecturally, the CLT should hold as soon as the expression for the limiting variance is finite, but most results on the CLT require significantly stricter regularity conditions. In this talk we will review recent progress on this conjecture.

Joint work with P. Sosoe.

HUGO LATOURELLE-VIGEANT, McGill
Matrix Dyson Equation for Correlated Linearizations

The exploration of large random matrices through asymptotic deterministic equivalents has been approached by a multitude of techniques. One approach employs the matrix Dyson equation to establish an asymptotic equivalence between a random resolvent and the solution of a matrix fixed point equation. Another, the linearization trick, has proven effective in studying rational functions of random matrices. This trick involves embedding a matrix expression into a larger random matrix, known as a linear matrix pencil, with a simplified correlation structure.

In this presentation, we introduce an extension of the matrix Dyson equation framework tailored specifically for linearizations. This extends previous work which has focused primarily on the case of pencils with blocks of canonical Wigner or Circular type. Within this framework, we derive an anisotropic global law for a broad class of pseudo-resolvents with general correlation structures. To highlight the practical implications of our framework, we apply it to a problem coming from machine learning. Specifically, we apply it to derive an exact asymptotic expression for the validation error of random features ridge regression and establish a general Gaussian equivalence result.

ANDRAS MESZAROS, University of Toronto
Eigenvectors of the square grid plus GUE

Eigenvectors of the GUE-perturbed discrete torus with uniform boundary conditions retain some product structure for small perturbations but converge to discrete Gaussian waves for large perturbations. We determine where this phase transition happens. Joint work with Balint Virag.

JAMES MINGO, Queen's UNiversity
Infinitesimal Operators

In this talk (which is joint work with Pei-Lun Tseng (NYU Abu Dhabi)) I will show that free independence can be adapted to give spectral results on finite rank perturbations of unitarily invariant matrix ensembles.

The main concepts are that of an infinitesimal operator (the finite rank perturbation) and infinitesimal free independence. Free independence is Voiculescu's adaptation of independence to non-commuting random variables based on free products, and infinitesimal independence is a stronger form of free independence. Free independence has become in recent years one of the main tools in analyzing the eigenvalue distribution of sums and products of random matrices.

I will apply this to the commutator and anti-commutator of independent operators.

VINCENT PAINCHAUD, McGill University
Convergence of the stochastic Airy operator to the stochastic sine operator

The Airy and sine point processes describe the behavior of eigenvalues of random matrices from beta-ensembles when scaled at the soft edge and at the bulk respectively. These two point processes can be described as the spectra of two stochastic differential operators called the stochastic Airy and sine operators. It is known that in a suitable scaling limit, the Airy point process converges in distribution to the sine point process. In this talk, we present an operator-level version of this convergence. More precisely, we represent the stochastic Airy and sine operators as random canonical systems, and we show that, when seen as measures, their coefficient matrices converge weakly in distribution. This talk is based on joint work with Elliot Paquette.

LUKE PEILEN, Temple University
Local Laws and Fluctuations for Log Gases

We study the statistical mechanics of the log-gas, or β-ensemble, for general potential and inverse temperature. By means of a bootstrap procedure, we prove local laws on a next order energy that are valid down to microscopic length scales. To our knowledge, this is the first time that this kind of a local quantity has been controlled for the log-gas. Simultaneously, we exhibit a control on fluctuations of linear statistics that is valid at all mesoscales. Using these local laws, we are able to exhibit for the first time a CLT at arbitrary mesoscales, improving upon a previous result of Bekerman-Lodhia that was true only for power mesoscales.

The approach that we use generalizes well to the study of Riesz gases in higher dimensions. Time permitting we will discuss some partial extensions of the above work to Riesz gases.

DAVID RENFREW, Binghamton University
Eigenvalues of minors of random matrices and roots of derivatives of random polynomials

I will describe the limiting behavior of the eigenvalues of minors of large bi-unitarily random matrices and the roots of derivatives of polynomials with independent, random coefficients, by giving a convolution semi-group which relates the two processes together. This is joint work with Andrew Campbell and Sean O'Rourke.

AARON SMITH, uOttawa and TIMC
Kac's Walk on SO(n) and Related Chains

Kac's walks on the sphere and on the special orthogonal group, introduced in 1953 and 1970, have long histories in the statistical physics and computational statistics literatures. I will describe the history of these walks and present a method for estimating their mixing times in terms of the singular values of a related random matrix. Time permitting, I'll introduce some closely-related but more complicated problems in random matrix theory that may be of interest to this workshop's participants.