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Double Category Sites for Grothendieck Topoi

In (Cahiers, 2020) DeWolf and I introduce a type of double categories with an Ehresmann topology as sites for étendues. This extends work by Lawson and Steinberg for ordered groupoids: we reinterpret them as double categories and establish a 2-adjunction between the 2-categories of Ehresmann sites and left-cancellative Grothendieck sites. However, since the category of left-cancellative sites does not satisfy the Ore condition for the class of Comparison Lemma maps, we cannot represent geometric morphisms between étendues as a bicategory of fractions for these sites.

In this talk I will introduce a generalization for the notion of Ehresmann site that can be used to represent any Grothendieck topos. The corresponding class of Grothendieck sites is that of the covering-mono sites: sites for which the single arrow coverages and the monics form an orthogonal factorization system. We show that the Comparison Lemma maps do give a calculus of fractions for this class of sites. The corresponding Ehresmann site double category has covering arrows as horizontal arrows and inclusions of subobjects as vertical arrows. We introduce a notion of covering-flat and covering preserving double functors between these Ehresmann sites with a class of Comparison Lemma double functors, that lets us express geometric morphisms in terms of fractions. This presentation restricts to étendues by only considering the so-called torsion-free generated sites. If time permits I will discuss how this can be used to represent orbifolds. This is joint work with Darien DeWolf (St Francis Xavier University) and Julia Ramos González (UC Louvain).