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Robustness for Near-Brownian Noise via Rough Paths Theory

One frequent modelling choice made in stochastic control is assuming that the noise is a Brownian motion. However, this is only an idealization. For example, Kushner argues that "wide-band" Brownian motion is much more physical because the high frequencies are often not present. Another example is the fractional Brownian motion which can be seen as the generalization of Brownian motion to allow for correlations in the increments. There are several other "near-Brownian" but not actually Brownian driving signals which lead to questions of robustness. That is - if we assume an idealized Brownian noise apply the optimal policy to a real situation, are we near optimal? We show that the answer is yes, using rough paths theory.

Rough paths theory was in invented in the 1990s by Terry Lyons as an alternative to the standard Itô theory. One of the issues with Itô theory is a lack of continuity in the driving noise. Another issue with Itô theory is that it only allows for semimartingale noise. This disallows noise such as fractional Brownian motion. One further issue is that Itô theory is not defined pathwise - only as limits in L^2. The key insight of rough paths theory is that by enhancing the driving signal with its "iterated integrals", continuity to the solution map is restored and a wide range of signals can be integrated.

In this talk, we introduce the basics of rough paths theory and discuss a robustness result. Joint with Somnath Pradhan and Serdar Yüksel.