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Exact solitary wave solutions for a coupled gKdV-NLS system

We study a coupled gKdV-NLS system $u_t + \alpha u^p u_x + \beta u_{xxx} = \gamma(|\psi|^2)_x$ and $i\psi_t + \kappa\psi_{xx} = \sigma u\psi$ with a general nonlinearity power $p > 0$, which has been introduced in the literature to model energy transport in anharmonic crystal materials. There is a strong interest in obtaining exact solutions describing frequency-modulated solitary waves $u = U(x - ct)$, $\psi = e^{i\omega t}\Psi(x - ct)$, where c is the wave speed, and ω is the modulation frequency. For the KdV case $p = 1$, some solutions are known, while for the mKdV case $p = 2$, no exact solutions have been found to-date, and nothing has been done for higher nonlinearities $p \geq 3$.

In the present work, we derive exact solutions for $p = 1, 2, 3, 4$, starting from the travelling wave ODE system satisfied by U and Ψ. The method is new: (i) obtain first integrals by use of multi-reduction symmetry theory; (ii) apply a hodograph transformation which leads to triangular (decoupled) system; (iii) introduce an ansatz for polynomial solutions of the base ODE; (iv) characterize conditions under which solutions yield solitary waves; (v) solve an algebraic system for the coefficients in the ansatz under those conditions. The resulting solitary waves exhibit a wide range of features: bright and dark peaks; single peaked and multi-peaked; zero and non-zero backgrounds.