Harmonic Analysis & PDE

(Org: **Ryan Gibara** (University of Cincinnati) and/et **Scott Rodney** (Cape Breton University))

ALEKSANDER DANIELSKI, Concordia University

Complex Analytic Structure of Stationary Solutions of the Euler Equations

This work is devoted to the stationary solutions of the 2D Euler equations describing the time-independent flows of an ideal incompressible fluid. There exists an infinite-dimensional set of such solutions. Prevous authors considered the solutions in the Frechet space of smooth functions and used powerful methods such as the Nash-Moser-Hamilton implicit function theorem to provide a smooth local parameterization of stationary flows in annular domains without stagnation point. However, in their approach they overlook a surprising feature of the stationary flows which makes the picture much more transparent, and opens the way to further progress. This is the observation that the flow lines are analytic curves, despite limited regularity of the velocity field.

To study the stationary flows we change the viewpoint and consider the flow field as a family of analytic flow lines non-analytically depending on parameter. We quantify the analyticity by introducing spaces of functions which have analytic continuation in one of the variables to a complex strip containing the real axis, with appropriate behaviour on the strip boundary. These partially-analytic functions form a complex Banach space. The stationary solutions satisfy (in the new coordinates) a quasilinear elliptic equation, degenerating in the presence of a stagnation point. Local solvability is proved by using the Banach analytic implicit function theorem. Thus we prove that the set of stationary flows is locally a complex analytic Banach manifold.

JOSHUA FLYNN, CRM/ISM, McGill University

LIOUVILLE-TYPE RESULT FOR THE CR YAMABE EQUATION IN THE HEISENBERG GROUP

We obtain a Liouville-type result for solutions to the CR Yamabe equation in the Heisenberg group H^n under a pointwise decay assumption at infinity, which extends a result obtained by Jerison and Lee for solutions in $L^{2+2/n}(H^n)$. Our proof relies on integral estimates combined with an extended version of a divergence formula by Jerison and Lee.

PAUL GAUTHIER, Université de Montréal

Radial limits of solutions to elliptic partial differential equations

For certain elliptic differential operators L, we study the behaviour of solutions to $Lu = 0$, as we tend to the boundary along radii in strictly starlike domains in $\mathbb{R}^n, n \geq 3$. Analogous results are obtained in other special domains. Our approach involves introducing harmonic line bundles as instances of Brelot harmonic spaces and approximating continuous functions by harmonic functions on appropriate subsets. These approximation theorems on harmonic spaces yield interesting examples for approximation by solutions of $Lu = 0$ on some domains in $\mathbb{R}^n$. Joint work with Mohammad Shirazi.

KIRILL GOLUBNICHIY, University of Calgary

Inverse Problem for the Black-Scholes Equation solution.

This new technique uses the Black-Scholes equation supplied by new intervals for the underlying stock and new initial and boundary conditions for option prices. The Black-Scholes equation was solved in the positive direction of the time variable, This ill-posed initial boundary value problem was solved by the so-called Quasi-Reversibility Method (QRM). This approach with an added trading strategy was tested on the market data for 368 stock options and good forecasting results were demonstrated. We use the geometric Brownian motion to provide an explanation of that effectivity using computationally simulated data for European call options. We also provide a convergence analysis for QRM. The key tool of that analysis is a Carleman estimate.

DAMIR KINZEBULATOV, Université Laval
An Orlicz space dictated by drifts singularities

In a recent paper with Semenov, we established an almost sharp result on weak solvability of singular SDEs. The result is sharp up to the strict inequality in the hypothesis on the magnitude of drift singularities (EJP, 2023). Nothing was known, even at the PDE level, about what happens in the critical case, i.e. when the magnitude of the singularities takes exactly the borderline value. It turns out that the answer lies within a certain Orlicz space. This Orlicz space is essentially dictated by the diffusion equation.

JOSH KLINE, University of Cincinnati
On regularity of sets of finite fractional perimeter in metric measure spaces

Federer's characterization states that a set is of finite perimeter if and only if its measure theoretic boundary has finite codimension 1 Hausdorff measure. In this talk, we discuss the extent to which an analog of this result holds for sets of finite s-perimeter, with $0 < s < 1$, in doubling metric measure spaces. Here the nonlocal s-perimeter is defined via a Besov seminorm, and as shown by Dávila in $\mathbb{R}^n$ and Di Marino and Squassina in the metric setting, recovers the perimeter of a set as $s \to 1^-$ under suitable rescaling. Time permitting, we will also consider a nonlocal minimization problem for the s-perimeter, as introduced by Caffarelli, Roquejoffre, and Savin in $\mathbb{R}^n$, and discuss regularity results for minimizers in the metric setting.

JAVAD MASHREGHI, Laval University
A Banach–Steinhaus type theorem

We introduce the notion of an asymptotically equicontinuous sequence of linear operators, and use it to prove the following result. If X, Y are topological vector spaces, if $T_n, T : X \to Y$ are continuous linear maps, and if D is a dense subset of X, then the following statements are equivalent: (i) $T_n x \to Tx$ for all $x \in X$, and (ii) $T_n x \to Tx$ for all $x \in D$ and the sequence (T_n) is asymptotically equicontinuous.

This is joint work with T. Ransford.

GIANGVUTHANH NGUYEN, Old Dominion University
Asymptotic expansion of a singular potential near the nematic-isotropic phase transition point in the Landau-de Gennes theory

The Landau-de Gennes theory is a type of continuum theory that describes nematic liquid crystal configurations in the framework of the Q-tensor order parameter. In the free energy, there is a singular bulk potential which is considered as a natural enforcement of a physical constraint on the eigenvalues of symmetric, traceless Q-tensors. In this talk we shall discuss some analytic properties related to this singular potential. More specifically, we study the asymptotic expansion of this singular potential (up to fourth order) near the nematic-isotropic phase transition point.

MARIA NTEKOUME, Concordia University
Critical well-posedness for the derivative nonlinear Schrödinger equation on the line

This talk focuses on the well-posedness of the derivative nonlinear Schrödinger equation on the line. This model is known to be completely integrable and L^2-critical with respect to scaling. However, until recently not much was known regarding the well-posedness of the equation below $H^{\frac{1}{2}}$. In this talk we prove that the problem is well-posed in the critical space on the line, highlighting several recent results that led to this resolution. This is joint work with Benjamin Harrop-Griffiths, Rowan Killip, and Monica Visan.

CINTIA PACCHIANO, University of Calgary
Regularity Results for Double Phase Problems on Metric Measure Spaces

In this talk, we present local and global higher integrability properties for quasiminimizers of a class of double-phase integrals characterized by non-standard growth conditions. We work purely on a variational level in the setting of a doubling metric measure space supporting a Poincaré inequality. The main novelty is the use of an intrinsic approach, based on a double-phase Sobolev-Poincaré inequality.

During the past two decades, a theory of Sobolev functions and first degree calculus has been developed in this abstract setting. A central motivation for developing such a theory has been the desire to unify the assumptions and methods employed in various specific spaces, such as weighted Euclidean spaces, Riemannian manifolds, Heisenberg groups, graphs, etc.

Analysis on metric spaces is nowadays an active and independent field, bringing together researchers from different parts of the mathematical spectrum. It has applications to disciplines as diverse as geometric group theory, nonlinear PDEs, and even theoretical computer science. This can offer us a better understanding of the phenomena and also lead to new results, even in the classical Euclidean case.

MICHAEL PENROD, The University of Alabama
Convolution Operators on Matrix Weighted Variable Lebesgue Spaces

The theory of matrix $\mathcal{A}_p$ weights has attracted considerable attention, beginning with the work of Nazarov, Treil, and Volberg in the 1990s. In this talk, we describe our work to extend this theory to the variable Lebesgue spaces. Generalizing matrix $\mathcal{A}_p$ to the variable exponent setting plays a crucial role.

David Cruz-Uribe, Kabe Moen, and Scott Rodney proved that given a matrix weight $W \in \mathcal{A}_p$ and a nice function $\phi \in C_c^\infty(\Omega)$, the convolution operator $\mathbf{f} \mapsto \phi * \mathbf{f}$ is bounded and approximate identities defined using ϕ converge. We extend the convergence of this convolution operator to matrix weighted variable Lebesgue spaces. As an application of our work, we prove a version of the classical H=W theorem for matrix weighted, variable exponent Sobolev spaces.

CRISTIAN RIOS, University of Calgary
The Moser method for infinitely degenerate equations

We implement the Moser iteration method to obtain boundedness and continuity of solutions to degenerate elliptic equations in which the ellipticity degenerates to infinite order. The degenerate nature of the problem allows the equation to offer a modest improvement for solutions measured in an Orlicz norm that grows slower than any power greater than one. This work is part of an ongoing collaboration with Lyudmila Korobenko, Eric Sawyer and Ruipeng Shen.

ERIC SAWYER, McMaster University
A Proof of the Fourier Restriction Conjecture

We prove the Fourier restriction conjecture. To prove the conjecture, we use frames for Lp consisting of smooth compactly supported Alpert wavelets having a large number of vanishing moments, along with sharp estimates on oscillatory integrals, as part of a two weight testing strategy using pigeonholing via the uncertainty principle.

LEONID SLAVIN, University of Cincinnati
Monotone rearrangement and Bellman functions for VMO with generalized Campanato norm

We consider VMO on an interval equipped with a Campanato-type norm and prove that monotone rearrangement does not increase the norm in this space. This allows us to compute Bellman functions for a family of integral functionals in this setting. Such functions are non-autonomous, in the sense that the length of the interval explicitly enters as one of the three Bellman variables, thus breaking with the method's traditional reliance on scale invariance. This is joint work with Pavel Zatitskii.

CODY STOCKDALE, Clemson University
On the $T1$ theorem for compactness of Calderón-Zygmund operators

We give a new formulation of the $T1$ theorem for compactness of Calderón-Zygmund singular integral operators. We prove that a Calderón-Zygmund operator T is compact on $L^2(\mathbb{R}^n)$ if and only if $T1, T^*1 \in \mathrm{CMO}(\mathbb{R}^n)$ and T is weakly compact. Compared to existing compactness criteria, our characterization more closely resembles David and Journé's classical $T1$ theorem for boundedness and follows from a simpler argument.

IGNACIO URIARTE-TUERO, University of Toronto
Some remarks on Muckenhoupt Ap weights

I will present some recently discovered remarks on Muckenhoupt Ap weights, and time permitting, will contextualize their use.

MAHISHANKA WITHANACHCHI, Laval University
Polynomial Approximation in Local Dirichlet Spaces

In this study, we investigate the behavior of partial Taylor sums, denoted as S_n, and Cesàro means (σ_n) within local Dirichlet spaces $(\mathcal{D}_\zeta)$, offering a comparative analysis with the classical disc algebra setting. Within the classical disc algebra $(\mathcal{A})$, the precise norm of S_n, commonly known as Lebesgue constants, remains indeterminate, displaying an asymptotic growth rate reminiscent of logarithmic behavior.

Within $\mathcal{D}_\zeta$, we explore various norm definitions, revealing distinct operator norm values for both S_n and σ_n. Our analysis unveils that for S_n, three specific norms exhibit a growth rate approximating $\sqrt{n}$ as n progresses. Notably, we also identify the existence of functions in $\mathcal{D}_\zeta$ for which the local sequence $\|S_n f\|_{\mathcal{D}_\zeta}$ diverges without bound. Furthermore, it is essential to emphasize that the norms associated with σ_n remain bounded within the context of $\mathcal{D}_\zeta$, highlighting a significant departure from the classical disc algebra setting.

JUNJIE ZHU, University of British Columbia
Cones are not Salem

The notions of Hausdorff and Fourier dimensions are ubiquitous in harmonic analysis and geometric measure theory. It is known that any hypersurface in $\mathbb{R}^{d+1}$ has Hausdorff dimension d. However, the Fourier dimension depends on the finer geometric properties of the hypersurface. For instance, the Fourier dimension of a hyperplane is 0, and the Fourier dimension of a hypersurface with non-vanishing Gaussian curvature is d. Recently, Harris has shown that the Euclidean light cone in $\mathbb{R}^{d+1}$ has Fourier dimension $d-1$, which leads one to conjecture that the Fourier dimension of a hypersurface equals the number of non-vanishing principal curvatures. We prove this conjecture for all d-dimensional cones in $\mathbb{R}^{d+1}$ generated by hypersurfaces in $\mathbb{R}^d$ with non-vanishing Gaussian curvature. In particular, cones are not Salem. Our method involves substantial generalizations of Harris's strategy.