MOKSHAY MADIMAN, University of Delaware

Submodularity questions in convex geometry

I will present some results, obtained in collaboration with M. Fradelizi, M. Meyer, and A. Zvavitch, on the log-submodularity of the volume of Minkowski sums on different classes of compact convex sets. The underlying question is the following: given a class $\mathcal{G}_n$ of compact convex sets in $\mathbb{R}^n$ closed under Minkowski summation and affine transformations, and $A, B, C \in \mathcal{G}_n$, what is the best constant α in the inequality

$$|A + B + C| \cdot |A| \leq \alpha |A + B| \cdot |A + C|,$$

where $|\cdot|$ denotes volume? This constant, which we may denote $\alpha(\mathcal{G}_n)$, has interesting interpretation, and it is particularly useful to identify cases where it is equal to 1, since in this case, it is intimately connected to inequalities for projections. If $\mathcal{K}_n$ is the collection of compact convex sets and $\mathcal{Z}_n$ is the collection of zonoids in $\mathbb{R}^n$, we show that $\alpha(\mathcal{K}_2) = \alpha(\mathcal{Z}_2) = 1$ and $1 = \alpha(\mathcal{Z}_3) < \alpha(\mathcal{K}_3) = 4/3$. We will also present some estimates and some conjectures for higher dimensions.