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*Four-class $Q$-bipartite association schemes*

A *(symmetric) association scheme* can be viewed as a real subalgebra $\mathbb{A}$ of an algebra of square matrices over the reals in which every element is symmetric, which is closed under entrywise multiplication $\circ$ and contains both $I$ and $J$ (the matrix of all ones). Let $E$ be a matrix in $\mathbb{A}$ and, for $0 \leq j \leq d = \dim \mathbb{A}$, denote by $E^{\circ j}$ the matrix whose entries are the $j^{\text{th}}$ powers of the entries of $E$. We say the association scheme is *$Q$-polynomial* (or *co-metric*) with *$Q$-polynomial generator* $E$ if the linear spans $\mathcal{I}_j = \langle J, E, \ldots, E^{\circ j} \rangle$ form a chain of ideals in $\mathbb{A}$ with $\mathcal{I}_d = \mathbb{A}$. It follows that $\mathbb{A}$ admits a vector space basis $E_0, E_1, \ldots, E_d$ with $E_i E_j = \delta_{i,j} E_i$ where $E_i$ is expressible as a polynomial of degree $i$ applied entrywise to $E$. In this talk, we focus on the *$Q$-bipartite* case where $(E_i \circ E_j)E_k = 0$ whenever $i + j + k$ is odd. We specialize Schoenberg's Theorem to this case and apply it to certain families with $d = 4$. The talk is mostly based on joint work with Brian Kodalen.