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Periodicity of Oriented Cayley Graphs

Let X be an oriented graph on n vertices. Its adjacency matrix A is skew-symmetric, i.e., if $u, v \in V(X)$ and uv is an arc in X, then $A_{u,v} = 1$ and $A_{v,u} = -1$, and if neither uv nor vu is an arc in X then $A_{u,v} = 0$. As A is skew-symmetric, iA is Hermitian and the matrices

$$U(t) := \exp(-tA), \quad (t \in \mathbb{R}),$$

are orthogonal. It follows that they determine a continuous quantum walk on ther vertices of X. We say a matrix is *flat* if its entries all have the same absolute value. A quantum walk admits *uniform mixing* at time t if $U(t)$ is flat.

If an $n \times n$ unitary matrix is flat, the absolute values of its entries are equal to $1/\sqrt{n}$, and if $U(t)$ is real and flat, then $\sqrt{n}U(t)$ is a real Hadamard matrix. This provides one reason why we are interested in uniform mixing. We have been trying to characterize which oriented Cayley graphs of abelian groups admit uniform mixing. We have proved that if a Cayley graph admits uniform mixing, the matrix-valued function $U(t)$ is periodic, and that this holds if and only if the eigenvalues of A are all integer multiples of $\sqrt{\Delta}$ for some (necessarily negative) square-free integer Δ. We are able to characterize the possible connection sets of these Cayley graphs.

In my talk I will discuss some of these results, and the machinery used to derive them. This is all joint work with Xiaohong Zhang.