KATE NIMEGEERS, University of Victoria

Pseudoku: A Sudoku Adjacency Algebra and Fractional Completion Threshold

We develop a 4-partite graph representation, G_P, for a partial Sudoku, P. The partite sets correspond to the rows, columns, boxes, and symbols of P. The edges represent unfulfilled conditions in P that are necessary for a completed Sudoku. For instance, if a symbol is missing from a row in P then an edge is drawn between those two vertices in G_P. We define a tile to be a 4-vertex subgraph of G_P corresponding to a valid placement of a symbol in P, noting that P can be completed if and only if G_P permits an edge-decomposition into tiles. We then relate the existence of such a decomposition to the existence of a solution to a specific linear system using an edge-tile inclusion matrix. Through an in-depth analysis of this matrix structure, we uncover a Sudoku adjacency algebra. This algebraic framework is constructed from a coherent configuration consisting of equivalence relations among row-column, row-symbol, column-symbol, and box-symbol Sudoku conditions.

The primary result we present is a minimum degree threshold for G_P that allows for a fractional tile-decomposition and therefore implies the existence of a fractional completion of P. The proof employs spectral decomposition, the properties of coherent configurations, and perturbation theory to estimate a generalized inverse for the matrix representation of a partial Sudoku puzzle in order to find a solution for the relaxed linear system. Improving on this result by finding a minimum degree threshold for an exact tile-decomposition is an interesting open question in this research area.