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Maximal determinants of matrices with entries in the roots of unity

Hadamard's determinant inequality states that the determinant of a complex matrix M of order n with entries taken from the complex unit circle satisfies $|\det M| \leq n^{n/2}$ — the matrix M meets this bound with equality if and only if $MM^{*} = nI_n$.

It is well known that when M is real (having ± 1 entries), then $MM^{\intercal} = nI_n$ implies that $n = 1, 2$ or n is a multiple of 4. Therefore, if we consider the maximal determinant of matrices with entries in the set $\{+1, -1\}$, then Hadamard's bound is not achievable at odd orders, or at orders $n \equiv 2 \pmod 4$ larger than 2. In the literature, one can find improved upper and lower bounds for the determinant of ± 1 matrices — the exact values of the maximal determinant have been determined for small values of n, and through several infinite families of examples.

In this talk, we consider the more general problem of finding the maximal determinant of matrices with entries taken from the set μ_m of m-th roots of unity, for some fixed value of m. We will present new upper and lower bounds for the determinant for a general value of m, and study the ternary case $m = 3$, and quaternary case $m = 4$ in more detail.

The maximality of the determinant sometimes imposes strong regularity conditions that make the matrices be equivalent to certain combinatorial designs. Conversely, constructions making use of designs or finite geometries, often give large values for the determinant. We will pay special attention to such connections.