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Divisible and transverse Bussey systems

A set-system of order N is a pair (X, B), where X is N-element set of points and B is a collection of subsets of X called $blocks$.

In 1852, Professor Dr. J. Steiner of Berlin, asked for which number N does there exist a set system containing no pairs that has order N and maximum block size k satisfying

(1) no block properly contains another block, and

(2) for all $t = 2, 3, ..., k-1$ every t-set that does not contain a block is contained in exactly one block of size $(t+1)$.

W.H. Bussey from the University of Minnesota in 1914 constructed the only known solution. His construction provided for each $k \geq 5$ a set-system of order $N = 2^{k-1} - 1$ and maximum block size k that satisfies Steiner's conditions. At the CMS 75th+1 anniversary summer meeting, I presented our investigation on this problem. See:

C.J. Colbourn, D.L. Kreher and P.R.J. Östergård, Bussey systems and Steiner's tactical problem. $Glas. Mat. Ser. III$, web.math.pmf.unizg.hr/glasnik/forthcoming/pGM7100.pdf

Today's discussion will examine what happens when pairs are allowed as blocks. In particular we consider as blocks the edges of the complete multipartite graph $G = K_{n_1, n_2, \ldots, n_r}$ or its complement $\overline{G}$.