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Exploring the Advantages of Using Sketched Krylov Methods in PRIMME

In recent years, sketching methods have increased in popularity for large scale least-squares problems. This is due to the scalability and reliability of randomized subspace embeddings which turn a large problem into something more managable with minimal loss of accuracy in the solutions. It had been observed that Rayleigh-Ritz approaches in Krylov iterative methods can be written as a least-squares problem, and therefore may benefit from the use of sketching to extract approximate solutions from the eigenspace.

One known pitfall of Krylov methods is that as the Krylov basis is being built, the condition number can grow exponentially due to numerical error and repeated directions occurring. Theoretically, we can reorthogonalize the basis using techniques such as Gram-Schmidt or the QR decomposition. However, in practice, these methods are expensive to execute and result in a computational bottleneck, particularly when looking for a large number of eigenpairs.

One benefit of using sketching in conjunction with Krylov methods is that we can avoid having to reorthogonalize our basis frequently. In theory, as long as the condition number of the basis remains below $\epsilon_{\texttt{mach}}$, sketched Rayleigh-Ritz should produce accurate Ritz vectors. Using the eigensolver software package PRIMME, we explore the benefits and disadvantages of using sketching with two popular Krylov methods, Lanczos and Davidson.