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Unravelling the Friedrichs Angle: A Key to Lower Bounds on the Minimum Singular Value

Estimating the eigenvalues of a sum of two symmetric matrices, say $P+Q$, in terms of the eigenvalues of P and Q, has a long tradition. To our knowledge, no study has yielded a positive lower bound on the minimum eigenvalue, $\lambda_{\min}(P+Q)$, when $P+Q$ is symmetric positive definite with P and Q singular positive semi-definite. We derive two new lower bounds on $\lambda_{\min}(P+Q)$ in terms of the minimum positive eigenvalues of P and Q. The bounds take into account geometric information by utilizing the Friedrichs angles between certain subspaces. The basic result is when P and Q are two non-zero singular positive semi-definite matrices such that $P+Q$ is non-singular, then $\lambda_{\min}(P+Q) \geq (1-\cos\theta_F)\min\{\lambda_{\min}(P), \lambda_{\min}(Q)\}$, where $\lambda_{\min}$ represents the minimum positive eigenvalue of the matrix, and θ_F is the Friedrichs angle between the range spaces of P and Q. We will discuss the interaction between the range spaces for some pair of small matrices to elucidate the geometric aspect of these bounds. Such estimates lead to new lower bounds on the minimum singular value of full rank 1×2, 2×1, and 2×2 block matrices in terms of the minimum positive singular value of these blocks. Some examples provided in this talk further highlight the simplicity of applying the results in comparison to some existing lower bounds. This is joint work with S. H. Lui (University of Manitoba).