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Toroidal integrals of Kudla-Millson forms and diagonal restrictions of Hilbert modular forms

Let Y be the locally symmetric spaces of an orthogonal group of signature (p,q). It is a Riemannian manifold of dimension pq and examples of such spaces include modular curves, Hilbert modular surfaces, Bianchi manifolds or more generally hyperbolic manifolds. The Kudla-Millson theta serie Θ_{KM} is a closed differential q-form on Y valued in a space of modular forms of weight $\frac{p+q}{2}$. By integrating this form on q-cycles in Y, it realizes a theta correspondence between the homology $H_q(Y)$ and this space of modular forms, often referred to as the *Kudla-Millson theta lift*. One of its most interesting features is that the Fourier coefficients of this lift can be expressed in terms of certain intersections numbers in Y

A very natural family of cycles is obtained by attaching a cycle $C_{\mathbf{T}}$ in $H_q(Y)$ to an algebraic tori $\mathbf{T}$ of the orthogonal group. In this talk, I will discuss the Kudla-Millson theta lift of such cycles, and in particular explain how the image of $C_{\mathbf{T}}$ is the diagonal restriction of a Hilbert modular forms of parallel weight one for $\mathrm{SL}_2(F_{\mathbf{T}})$, where $F_{\mathbf{T}}$ is a totally real étale algebra attached to $\mathbf{T}$. In the case of signature $(2,2)$, one can recover a result of Darmon-Pozzi-Vonk about the diagonal restriction of Eisenstein series, as well as a *trace identity* due to Darmon-Harris-Rotger-Venkatesh.