A celebration in honor of Jean-Marie De Koninck's 75th birthday: Elementary and Analytic Number Theory
Théorie élémentaire et analytique des nombres : Une célébration à l'honneur du 75ème anniversaire de Jean-Marie de Koninck
(Org: **Nicolas Doyon** and/et **William Verreault** (Université Laval))

THEOPHILUS AGAMA, Université Laval
On the joint work of Jean-Marie De Koninck and Imre Kátai

In this talk, I will present highlights of the joint work of Jean-Marie De Koninck and Imre Kátai produced over the past 40 years, putting much emphasis on their contribution to the study of normal numbers.

HUGO CHAPDELAINE, Université Laval
Conditional convergence in the critical strip for lattice zeta functions associated to totally real fields

The goal of this talk is to explain how a miraculous formula of Brion, related to the enumeration of lattice points in integral convex polytopes implies the conditional convergence of certain Dirichlet series $Z(s)$ when the complex parameter s is such that $1-\epsilon < Re(s)$, for ϵ small enough. Note that the order of summation of the series $Z(s)$ is defined in a geometrical way. In order to simplify the presentation we shall focus on the simplest non-trivial case namely when $Z(s)$ is a lattice zeta function associated to a real quadratic field K. In that case one can take $\epsilon = \frac{1}{2}$.

CHI HOI, University of British Columbia
Diophantine tuples over integers and finite fields

A set $\{a_1, a_2, \ldots, a_m\}$ of distinct positive integers is a Diophantine m-tuple if the product of any two distinct elements in the set is one less than a square. There is a long history and extensive literature on the study of Diophantine tuples and their generalizations in various settings. In this talk, we focus on the following generalization: for each $n \geq 1$ and $k \geq 2$, we call a set of positive integers a Diophantine tuple with property $D_k(n)$ if the product of any two distinct elements is n less than a k-th power, and we denote $M_k(n)$ be the largest size of a Diophantine tuple with property $D_k(n)$. Using various tools from number theory, we show that there is $k = k(n)$ such that $k, n \to \infty$ and $M_k(n) = o(\log n)$, breaking the $\log n$ barrier. A key ingredient is to study the finite field model of the same problem. Joint work with Seoyoung Kim and Semin Yoo.

OMAR KIHEL, Brock University
On the index of a number field and some connected open questions

Let K be a number field of degree n over $\mathbb{Q}$, and A its ring of integers. Let $I(K) = \gcd\{[A : Z[\alpha]]\}$, where α primitive element of A. Let p a prime and $I_p(K)$, be the $p-$adic valuation of $I(K)$. Determining $I_p(K)$ is still an open problem. In this talk we will report on some progress towards the computation of the prime divisors of $I(K)$ and some other connected questions.

SUN-KAI LEUNG, Université de Montréal
Central limit theorems for arithmetic functions in short intervals

The distribution of arithmetic functions is a central topic in analytic number theory. In this talk, we discuss various central limit theorems for arithmetic functions in short intervals, in which the Fourier duality lies at the heart of the matter.

FLORIAN LUCA, Wits University
On the index of friability

In a collaboration that spans for 20 years, Jean-Marie De Koninck and the speaker have published 26 research papers, co-authored one textbook and co-edited a book on Anatomy of Integers. In my talk, I will survey a few of our joint results. I will also present some work in progress concerning the index of friability of integers.

SIVA NAIR, Université de Montréal
The Mahler measure of some polynomial families

The Mahler measure of a polynomial $P(x_1, x_2, \ldots, x_n)$ is the average value of $\log|P|$ along the unit n-torus $\mathbb{T}^n$ defined by $|x_i| = 1$ for all i. Interest in this quantity arose from the fact that the Mahler measure of certain polynomials is quite remarkable and not just any random real number – they evaluate to special values of L-functions! However, in general, it is very difficult to evaluate Mahler measures of multivariable polynomials. In this talk, we will consider some families of polynomials that contain, for every integer $n > 1$, an n-variable polynomial. We will discuss how the structure of these polynomials lets us compute their Mahler measures as combinations of values of the Riemann zeta function and values of certain Dirichlet L-functions. This talk includes joint work with Matilde Lalín and Subham Roy.

ARTHUR BONKLI RAZAFINDRASOANAIVOLALA, Université Laval
Integers with a sum of co-divisors yielding a square

Finding elliptic curves with high ranks has been the focus of much research. Recently, with the goal of generating elliptic curves with a large rank, some authors used large integers n which have many divisors, amongst which one can find divisors d such that $d + n/d$ is a perfect square. This strategy is in itself a motivation for studying the function $\tau_\square(n)$ which counts the number of divisors d of an integer n for which $d + n/d$ is a perfect square. We show that $\sum_{n \le x} \tau_\square(n) = c_\square x^{3/4} + O(\sqrt{x})$ for some explicit constant $c_\square$. Moreover, letting $\rho_1(n) := \max\{d \mid n : d \le \sqrt{n}\}$ and $\rho_2(n) := \min\{d \mid n : d \ge \sqrt{n}\}$ stand for the middle divisors of n, we show that the order of magnitude of the number of positive integers $n \le x$ for which $\rho_1(n) + \rho_2(n)$ is a perfect square is $x^{3/4}/\log x$. This is joint work with Jean-Marie De Koninck and Hans Schmidt Ramiliarimanana.

CIHAN SABUNCU, Université de Montréal
On the moments of the number of representations as sums of two prime squares

The solutions to the system of equations $x_1^2 + x_2^2 = x_3^2 + x_4^2$ with $x_i \le R$ in integers come in two pairs, the diagonal ones and the off-diagonal ones. The number of off-diagonal solutions is more than that of diagonal solutions. If we instead focus our attention to the case x_i are prime, then the diagonal solutions overtake the off-diagonal solutions; this effect is called "paucity". This phenomenon also continues in the case of the system of three equations. For more equations, we expect off-diagonal solutions to be the main contribution.
In this talk, we will give ideas on how to get an upper bound for the off-diagonal solutions in the two equations case. The approach we present is generalizable to many equations. If time permits, we will also see how to get lower bounds conditional on some quadratic extension of the Green-Tao theorem.

WILLIAM VERREAULT, University of Toronto
On the tower factorization of integers

I will report on recent (and fun!) joint work with Jean-Marie De Koninck on the factorization of integers into towers of primes. Writing an integer n as a product of prime powers p^a, then factoring each exponent a as a product of prime powers q^b, and so on, we obtain the *tower factorization* of n. We then study the *height* of an integer, namely the number of "floors" in its tower factorization. In particular, given a fixed integer $k \ge 1$, we will see a formula for the density of the set of integers with height k.

GARY WALSH, University of Ottawa - Tutte Institute
Powerful Numbers, Elliptic Curves and other Keywords

In a series of papers, Erdos posed numerous problems about powerful numbers, and more generally, k-full numbers. We will discuss two of these problems that can be solved by exploiting the group structure of elliptic curves defined over the rational numbers, the first of which being related to work of J.-M. de Koninck, and the second concerning the existence of quadruples of coprime powerful numbers in arithmetic progression. In particular, our computations suggest that a new example, which involves integers having 110 digits, is the smallest such quadruple. This is joint work with Michael Bennett.