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Imperfect and Bogdanov-Takens Bifurcations in Biological Models: From Harvesting of Species to Removal of Infectives

The bifurcation induced by small perturbations of a system (on already existed bifurcations) that leads to more complex bifurcations simplifying into separate saddle and node equilibrium points is called an imperfect (perturbed) bifurcation. In this paper, we examine two types of biological models that Fred Brauer made pioneer contributions: predator-prey models with stocking/harvesting and epidemic models with immigration/isolation. First we consider a predator-prey model with Holling type II functional response whose dynamics and bifurcation are well-understood. We will show that that introducing constant stocking/harvesting of predators induces imperfect bifurcation: For the case with stocking, the model has one positive equilibrium and one negative equilibrium when stocking constant increases from zero. For the case of harvesting, the model has none, one, or two positive equilibria when harvesting constant varies; then we explain that the unique positive equilibrium is a cusp of codimension 2 and the model undergoes Bogdanov-Takens bifurcation. We also consider an epidemic model with constant importation/isolation of infective individuals and observe similar imperfect and Bogdanov-Takens bifurcations when the constant perturbation rate varies. (Based on a joint work with Dongmei Xiao).