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Borel's conjecture and meager-additive sets

Strong measure zero sets were introduced by Borel and have been studied since the beginning of the previous century. Borel conjectured that every strong measure zero set of real numbers must be countable. A few years later, Sierpiński proved that if the continuum hypothesis (CH) is assumed, then there exists an uncountable strong measure zero set of reals. Nevertheless, the question about the relative consistency of Borel's conjecture remained open until 1976 when Laver, in a ground-breaking paper, constructed a model of set theory in which every strong measure zero set of reals is countable.

A result of Galvin, Mycielski, and Solovay provides a characterization of Borel's strong nullity in terms of an algebraic (or translation-like) property for subsets of the real line. By means of this characterization, a strengthening of strong nullity, meager-additivity, appeared on the scene. Meager-additivity and other smallness notions on the real line have received considerable attention in recent years. A 1993 question due to Bartoszyński and Judah asks whether strong nullity and meager-additivity have a very rigid relationship, in the following sense:

Question (Bartoszyński–Judah, 1993): Suppose that every strong measure zero set of reals is meager-additive. Does Borel's conjecture follow?

We proved that it is relatively consistent with ZFC that every strong measure zero subset of the real line is meager-additive while there are uncountable strong measure zero sets (i.e., Borel's conjecture fails), giving a negative answer to the question above.