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The Angle Process in Deep Neural Networks and the Bessel Numbers of the Second Kind

A mysterious property of deep neural networks is that, on initialization, the inputs tend to get more and more correlated as the network gets deeper and deeper. In this talk, we investigate fully connected networks with the ReLU non-linearity, and we discover how the angle between any two inputs evolves as a function of network depth. The formula involves the joint moments of the ReLU function applied to Gaussian random variables. We take a combinatorial approach to explicitly solve for these joint moments and doing so reveals a surprising connection to the Bessel numbers of the second kind. We are able to accurately predict the joint distribution of each layer on initialization given only the inputs into the network. The formula becomes more exact as the width of each layer tends to infinity. Both the mathematical theory behind the formula as well as simulations to validate our results are presented.