**Student Research Talks**
**Session de présentations des étudiants**
(Org: **Alexander Clow** and/et **Daniel Zackon**)

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**ALEXANDER CLOW**, Simon Fraser University
*An Introduction to Graph Colouring with Randomness*

Graph colouring is a classical field in graph theory and its applications that remains a central area of study in cutting edge research. This talk will introduce and motivate proper vertex colouring and some of its variants with a focus on how probabilistic techniques can be used to study these parameters. In particular we will describe the basic probabilistic method, then show several distinct ways that it can be applied to various graph colouring problems.

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**PINGPING CONG**, Western University
*Dynamics of a three-species food chain model with fear effect*

In this paper a three-species food chain model is formulated to investigate the impact of fear. First, we derive the predator's functional response by using the classical Holling's time budget argument and formulate a three-species food chain model where the cost and benefit of anti-predator behaviours are included. Then we study the dissipativity of the system and perform analysis on the existence and stability of equilibria. At last, we use numerical simulations to more visually explore the effects of fear on three species. The results show that the predator's fear effect can transform the system from chaotic dynamics to a stable state. Our results may provide some useful biological insights into ecosystems containing predator-prey interactions.

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**JENNY LAWSON**, University of Calgary
*Optimality and Sustainability of Delayed Impulsive Harvesting*

Optimal and sustainable management of natural resources requires knowledge about the behaviour of mathematical models of harvesting under many different types of conditions. In this talk, we will be investigating the sustainability and optimality of delayed impulsive harvesting. Impulses describe an instantaneous change in a system due to some external effect (like harvesting in a fishery), which has a duration that is negligible compared to the overall time scale of the process. These impulses can then be combined with differential equations (DEs) to form impulsive DEs.

Delays within harvesting can represent a dependency on information that is out of date. Since it is likely that most data used to make harvesting decisions will be at least somewhat out of date, including delays within impulsive conditions is a topic of current interest. A close connection to the dynamics of high-order difference equations is used to conclude that while the inclusion of a delay in the impulsive condition does not impact the optimality of the yield, sustainability may be highly affected and is once again delay-dependent. Maximum and other types of yields are explored, and sharp stability tests are obtained for the model. It is also shown that persistence of the solution is not guaranteed for all positive initial conditions, and extinction in finite time is possible, which provides a possible explanation for observed but unforeseen population collapses. Overall, the results imply that delays within harvesting should be kept short to maintain the sustainability of resources.

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**GUSTAVO CICCHINI SANTOS**, Toronto Metropolitan University
*Understanding Non-Equilibrium Steady States*

Physical systems are characterized by their response to perturbations. The Fluctuation Dissipation Theorem predicts the behavior of systems in equilibrium. Can an expression be derived using methods from quantum field theory to describe the vertex response to a perturbation, and is the Fluctuation Dissipation Theorem modified as a result of these perturbations. Using Berezin integration and properties of determinants we derive said expression. The derivation yields the same result as the less rigorous methods. We learn the Fluctuation Dissipation Theorem has an equilibrium-like response to a vertex perturbation making the Fluctuation Dissipation theorem a bad indicator of whether a system is in equilibrium or out of equilibrium.

**SILAS VRIEND**, McMaster University

*Infinite bubbles: a planar isoperimetric problem with two unbounded chambers*

The dual of the classical isoperimetric problem asks which planar curve (if any) minimizes perimeter among all curves enclosing a fixed area A. The answer, perhaps unsurprisingly, is a circle. The resultant geometric figure consists of one compact chamber (the interior of the circle) and one unbounded chamber (the exterior of the circle). In this talk, we consider the generalization to more than one unbounded chamber. Furthermore, we present a classical variational solution to a simplified version of the following problem: given a fixed area A to enclose, which (if any) is the locally perimeter-minimizing configuration among all partitions of the plane into one compact chamber and two unbounded chambers?