WILL DANA, University of Michigan
Walls of shards and filtrations of shard modules

The representation theory of preprojective algebras is intertwined with the combinatorics of Coxeter groups. In particular, for a simply laced Dynkin diagram, the King stability domains of bricks of its preprojective algebra partition the reflecting hyperplanes of the associated Coxeter group into cones called shards, which previously arose in work of Nathan Reading on the lattice structure of the weak order. In joint work with David Speyer and Hugh Thomas, we extend this result to non-Dynkin diagrams by showing a bijection between rigid bricks with full-dimensional stability domain (which we call "shard modules") and shards.

In this talk, we'll give an overview of this generalization, and then showcase a couple of ways in which the geometry of shards relates to properties of shard modules. We'll generalize a result from the Dynkin case, showing that, at any wall of a shard, it's met by two other shards such that the three corresponding shard modules fit into a nice short exact sequence. We'll then apply this result to families of diagrams with "tails" to visualize how shard modules are related to the positions of their shards in this case.