CMS Student Poster Session
Présentations par affiches des étudiants - SMC

STÉPHANIE ABO, University of Waterloo
Can the clocks tick together despite the noise? Stochastic simulations and analysis of the mean-field limit

The suprachiasmatic nucleus (SCN), also known as the circadian master clock, consists of a large population of oscillator neurons. Together, these neurons produce a coherent signal that drives the body's circadian rhythms. What properties of the cell-to-cell communication allow the synchronization of these neurons, despite a wide range of environmental challenges such as fluctuations in photoperiods? To answer that question, we present a mean-field description of globally coupled neurons modeled as Goodwin oscillators with standard Gaussian noise. Provided that the initial conditions of all neurons are independent and identically distributed, any finite number of neurons becomes independent and has the same probability distribution in the mean-field limit, a phenomenon called propagation of chaos. This probability distribution is a solution to a Vlasov-Fokker-Planck type equation, which can be obtained from the stochastic particle model. We study, using the macroscopic description, how the interaction between external noise and intercellular coupling affects the dynamics of the collective rhythm, and we provide a numerical description of the bifurcations resulting from the noise-induced transitions. Our numerical simulations show a noise-induced rhythm generation at low noise intensities, while the SCN clock is arrhythmic in the high noise setting. Notably, coupling induces resonance-like behavior at low noise intensities, and varying coupling strength can cause period locking and variance dissipation even in the presence of noise.

MARYAM ALHAWAJ, University of Toronto
Generalized pseudo-Anosov Maps and Hubbard Trees

The Nielsen-Thurston classification of the mapping classes proved that every orientation preserving homeomorphism of a closed surface, up to isotopy is either periodic, reducible, or pseudo-Anosov. Pseudo-Anosov maps have particularly nice structure because they expand along one foliation by a factor of $\lambda > 1$ and contract along a transversal foliation by a factor of $\frac{1}{\lambda}$. The number λ is called the dilatation of the pseudo-Anosov. Thurston showed that every dilatation λ of a pseudo-Anosov map is an algebraic unit, and conjectured that every algebraic unit λ whose Galois conjugates lie in the annulus $A_\lambda = \{z : \frac{1}{\lambda} < |z| < \lambda\}$ is a dilatation of some pseudo-Anosov on some surface S.

Pseudo-Anosovs have a huge role in Teichmuller theory and geometric topology. The relation between these and complex dynamics has been well studied inspired by Thurston.

In this project, I develop a new connection between the dynamics of quadratic polynomials on the complex plane and the dynamics of homeomorphisms of surfaces. In particular, given a quadratic polynomial, we show that one can construct an extension of it which is generalized pseudo-Anosov homeomorphism. Generalized pseudo-Anosov means the foliations have infinite singularities that accumulate on finitely many points. We determine for which quadratic polynomials such an extension exists. My construction is related to the dynamics on the Hubbard tree which is a forward invariant subset of the Julia set that contains the critical orbit.

CINDY CHEN, University of Saskatchewan
SIR Infectious Disease Modelling with Vaccination

The new coronavirus attacked the world in 2019, causing harm to people's lives and society in multiple aspects. It is therefore of high importance to develop reliable mathematical models that would be able to predict the development of similar pandemics under different scenarios, including vaccination strategies, to help inform governments and health care systems and facilitate optimal policy making.

In this work, we study an SIR ("Susceptible-Infected-Recovered") epidemic model that considers the time evolution of the three respective groups of population. Transitions between Susceptible, Infected, and Recovered groups are usually defined by constant coefficients, such as infection and recovery rates. The novel aspect of our model is making the coefficients time-dependent. This allows a significantly larger freedom in building the models and predicting the outcomes under different scenarios. As an example we choose the model coefficients to reflect a situation when, at a certain time, a vaccine is introduced. In this situation, it is shown that under the same parameters, vaccination leads to a significantly faster transition to a recovered population.

K. BHASKARA, A. COOK, McMaster University
Hadamard Product and Binomials Ideals

We study the Hadamard product of two varieties V and W, with particular attention to the situation when one or both of V and W is a binomial variety. The main result of this paper shows that when V and W are both binomial varieties, and the binomials that define V and W have the same binomial exponents, then the defining equations of $V \star W$ can be computed explicitly and directly from the defining equations of V and W. This result recovers known results about Hadamard products of binomial hypersurfaces and toric varieties. Moreover, as an application of our main result, we describe a relationship between the Hadamard product of the toric ideal I_G of a graph G and the toric ideal I_H of a subgraph H of G. We also derive results about algebraic invariants of Hadamard products: assuming V and W are binomial with the same exponents, we show that $\deg(V \star W) = \deg(V) = \deg(W)$ and $\dim(V \star W) = \dim(V) = \dim(W)$. Finally, given any (not necessarily binomial) projective variety V and a point $p \in \mathbb{P}^n \setminus \mathbb{V}(x_0 x_1 \cdots x_n)$, subject to some additional minor hypotheses, we find an explicit binomial variety that describes all the points q that satisfy $p \star V = q \star V$.

JENNY LAWSON, University of Calgary
Optimality and Sustainability of Delayed Impulsive Harvesting

Optimal and sustainable management of natural resources requires knowledge about the behaviour of mathematical models of harvesting under many different types of conditions. In this talk, we will be investigating the sustainability and optimality of delayed impulsive harvesting. Impulses describe an instantaneous change in a system due to some external effect (like harvesting in a fishery), which has a duration that is negligible compared to the overall time scale of the process. These impulses can then be combined with differential equations (DEs) to form impulsive DEs.

Delays within harvesting can represent a dependency on information that is out of date. Since it is likely that most data used to make harvesting decisions will be at least somewhat out of date, including delays within impulsive conditions is a topic of current interest. A close connection to the dynamics of high-order difference equations is used to conclude that while the inclusion of a delay in the impulsive condition does not impact the optimality of the yield, sustainability may be highly affected and is once again delay-dependent. Maximum and other types of yields are explored, and sharp stability tests are obtained for the model. It is also shown that persistence of the solution is not guaranteed for all positive initial conditions, and extinction in finite time is possible, which provides a possible explanation for observed but unforeseen population collapses. Overall, the results imply that delays within harvesting should be kept short to maintain the sustainability of resources.

LAILA MAHRAT, Lewis University
An Agent-Based Model of Environmental Transmission of Clostridioides difficile in Healthcare Settings

Clostridioides difficile (*C. difficile*) is one of the most frequently identified healthcare-acquired infections in United States hospitals. Colonized patients, both symptomatic and asymptomatic, shed *C. difficile* endospores that can survive for long periods on surfaces outside the host and are resistant to many commonly-used disinfectants. Transmission pathways can include contact with both endospores on fomites, objects likely to carry infection, and endospore-carrying individuals. Our agent-based model simulates the spread of *C. difficile* within a hospital ward, focusing on transmission originating from environmental pathways and healthcare workers. Simulations can help determine effective control strategies to mitigate the spread of *C. difficile* in healthcare settings.

ANA MUCALICA, McMaster University
Solitons on the rarefaction wave background via the Darboux transformation

Rarefaction waves and dispersive shock waves are generated from the step-like initial data in many nonlinear evolution equations including the classical example of the Korteweg-de Vries (KdV) equation. When a solitary wave is injected on the step-like initial data, it is either transmitted over or trapped inside the rarefaction wave background. We show that the transmitted soliton can be obtained by using the Darboux transformation for the KdV equation. On the other hand, we show with the help of numerical simulations that the trapped soliton disappears in the long-time dynamics of the rarefaction wave.

GAVIN OROK, University of Waterloo
Determining where Monte Carlo Outperforms Quasi-Monte Carlo for Functions Monotone in Each Coordinate in Dimensions 3 and Above

The Quasi-Monte Carlo methods are one way to estimate the integrals of functions over high-dimensional cubes. They are a variation of standard Monte Carlo methods; instead of choosing random points inside the cube to calculate an estimate of the result, Quasi-Monte Carlo scrambles a deterministic set of points that are sufficiently uniform inside of the cube. This is often desirable as it limits gaps and clusters of points that can harm the quality of the estimate.

One problem of interest to researchers of Quasi-Monte Carlo is to determine cases where these methods will outperform standard Monte Carlo methods, by having a lower theoretical variance in the final result. Previous work by Lemieux and Wiart showed that for two-dimensional functions monotone in each coordinate, Quasi-Monte Carlo will always outperform Monte Carlo in this way.

In this presentation, we will consider the extension of this problem to functions monotone in each coordinate in dimensions three and above. First, using computer searches we will find cases in higher dimensions where Monte Carlo has a lower theoretical variance than Quasi-Monte Carlo. Then, we will extend these cases to higher dimensions and determine relationships between them using equivalence classes and translations defined on sets of vectors called antichains.

KEVIN MIN SEONG PARK, University of Toronto
Deep Reinforcement Learning for Viscous Incompressible Flow

Numerical methods for approximating the solution to the incompressible Navier-Stokes equations typically solve discretized equations on a finite mesh of the domain, a computationally expensive process. We present a mesh-free method which can be easily scaled to irregular 3D geometries as we encode the domain and boundary through signed distance functions. The numerical solution is provided by a deep neural network trained on an objective that is derived from the expectation of a martingale stochastic process of the viscous Burgers equation, similar to Monte Carlo methods through the Feynman-Kac formula. We adopt a reinforcement learning paradigm of iterating the optimization step at every simulated increment of the Itô process. The vector potential is encoded into the neural network architecture, thereby automatically satisfying the incompressibility condition without requiring the pressure term. Simulation of the Itô process requires the true velocity, which we replace with the current approximation during the training procedure and we prove that this process is a fixed-point iteration in a simplified setting. This method is capable of numerically solving solutions to elliptic and parabolic partial differential equations. Deep learning is parallelizable and hyperparameters can be incorporated to solve a family of problems. We provide an example of flow past disk with a range of input flow speeds and viscosities, all provided by a single neural network, to highlight these advantages.

KALEB D. RUSCITTI, McGill University
The Verlinde formula for flat SU(2) connections using a toric degeneration

The moduli space M of flat SU(2) connections has a prequantum line bundle L and a polarization, the data required for geometric quantization. Jeffrey and Weitsman have shown the moduli space M of flat SU(2) connections has Hamiltonian functions which almost exhibit M as a toric variety. If it were toric, the theory of toric varieties tells us that the space of global sections of L, which is the quantum data, has dimension computed by the Verlinde formula. Hurtubise and Jeffrey have constructed a "master space" P with both a symplectic and a holomorphic description, which is toric and should contain all the information of M. Holomorphically, P is a space of framed parabolic sheaves over a punctured Riemann surface, and by degenerating the original Riemann surface to the punctured one, the moduli space M degenerates to the master space P. The aim now is to see how the recent work of Harada, Kaveh and Khovansky makes rigorous the justification of the Verlinde formula obtained by point counting by Jeffrey and Weitsman, hence giving a new proof of the formula.

KATARINA SACKA, McMaster University
Applications of Next-Iterate Operators to Discrete Planar Maps.

Two applications of next-iterate operators for discrete planar maps defined in the work by S.H. Streipert and G.S.K. Wolkowicz are explored. The time-delay equation

$$x_{n+1} = \frac{\alpha + x_{n-1}}{A + x_n}$$

for $n \in \mathbb{N}$, $\alpha \geq 0$, $A \in [0,1)$, $x_0 > 0$, and $x_1 > 0$ has a unique positive equilibrium which is a saddle point. Applying the change of variables, $y_n = x_{n-1}$, we write this equation as the planar system,

$$x_{n+1} = \frac{\alpha + y_n}{A + x_n}, \quad y_{n+1} = x_n.$$

We show that there exists a nontrivial positive solution which decreases monotonically to the equilibrium, proving Conjecture 5.4.6 from M. Kulenovic and G. Ladas. By using the augmented phase plane with nullclines and their associated root-curves, we can show the general behaviour of solutions in the plane. Using the tangent vector to the stable manifold at the equilibrium, we can show that solutions in a particular region defined by the nullclines and their associated root-curves, will decreases monotonically to the equilibrium along the tangent vector to the stable manifold. While Conjecture 5.4.6 has been previously proven, our proof provides a more elementary solution.

The second application of next-iterate operators regards the time delay equation,

$$x_{n+1} = \frac{\alpha + x_n + x_{n-1}}{A + x_n + x_{n-1}}$$

for $n \in \mathbb{N}$, $A > \alpha > 0$, $x_0 > 0$, and $x_1 > 0$. This equation has a unique positive equilibrium which is locally stable. Using the same change of variables as before, $y_n = x_{n-1}$, we write this equation as the planar system,

$$x_{n+1} = \frac{\alpha + x_n + y_n}{A + x_n + y_n}, \quad y_{n+1} = x_n.$$

By applying the augmented phase portrait, in addition to two new next-iterate operators defined in this work, we can expand this result to prove global stability.

DAYANNA SANCHEZ, Lewis University
Analyzing the Impact of Alternative Assessments and Growth Mindset

Alternate assessment techniques such as mastery grading, specifications grading, and standards-based grading are assessment techniques professors are implementing in order to support a growth mindset of learning. This proposal will support a multi-institutional collaboration that studies the impact of mastery grading assessment techniques on the growth mindset of students in a variety of mathematics classes. By analyzing pre- and post-surveys with questions adapted from Dweck's Mindset survey, we will explore whether there is a difference in the growth mindset between various cross-sections of student populations between classes (mastery and non-mastery, specific courses, universities, etc.) and whether the growth mindset of students changed by the end of the semester. This research will explore whether there is a difference in students' mindset of learning mathematics between various cross-sections of student populations between classes (mastery and non-mastery, specific courses, universities, etc.) and whether the growth mindset of students changed by the end of the semester.

GUSTAVO CICCHINI SANTOS, Toronto Metropolitan University
UNDERSTANDING NON-EQUILIBRIUM STEADY STATES

Physical systems are characterized by their response to perturbations. The Fluctuation Dissipation Theorem predicts the behavior of systems in equilibrium. Can an expression be derived using methods from quantum field theory to describe the vertex response to a perturbation, and is the Fluctuation Dissipation Theorem modified as a result of these perturbations. Using Berezin integration and properties of determinants we derive said expression. The derivation yields the same result as the less rigorous methods. We learn the Fluctuation Dissipation Theorem has an equilibrium-like response to a vertex perturbation making the Fluctuation Dissipation theorem a bad indicator of whether a system is in equilibrium or out of equilibrium. We then apply our result to a biochemical problem.

MELISSA MARIA STADT, University of Waterloo
Impact of feedforward and feedback controls on potassium homeostasis: Mathematical modelling and analysis

Dysregulation of potassium is a common and dangerous side effect of many pathologies and medications. Potassium homeostasis is primarily mediated by (i) uptake of potassium into the cells via the sodium-potassium pump and (ii) renal regulation of urinary potassium excretion. Due to the importance of potassium in cellular function and the daily challenge of undergoing variations in potassium intake, mammals have evolved several regulatory mechanisms to ensure proper potassium balance between the extra- and intracellular fluids. The multitude of physiological processes involved in potassium regulation makes its study well suited for investigation with mathematical modelling. In this project, we developed a compartmental model of extra- and intracellular potassium regulation. We included a detailed kidney compartment with the effects of aldosterone and potassium intake on renal potassium handling as well as intracellular potassium uptake stimulation by both insulin and aldosterone. Model simulations were conducted and analyzed to quantify the impact of individual regulatory mechanisms on whole-body potassium regulation. Additionally, we used this model to simulate and give evidence for a newly hypothesized signal, muscle-kidney cross talk, on potassium loading and depletion.

YUN-CHI TANG, University of Toronto
On Knots That Divide Ribbon Knotted Surfaces

We define a knot to be half ribbon if it is the cross-section of a ribbon 2-knot, and observe that ribbon implies half ribbon implies slice. We introduce the half ribbon genus of a knot K, the minimum genus of a ribbon knotted surface of which K is a cross-section. We compute this genus for all prime knots up to 12 crossings, and many 13-crossing knots. The same approach yields new computations of the doubly slice genus. We also introduce the half fusion number of a knot K, that measures the complexity of ribbon 2-knots of which K is a cross-section. We show that it is bounded from below by the Levine-Tristram signatures, and differs from the standard fusion number by an arbitrarily large amount.

WILLIAM VERREAULT, Université Laval
Series expansion via unwinding

We present a general unwinding scheme for analytic functions as well as convergence theorems for the unwinding series expansion, extending results on the Blaschke unwinding series, a nonlinear analogue of Fourier series with a wide range of practical applications.

YUMING ZHAO, University of Waterloo
There is no sum-of-squares certificate for positivity in tensor product of free algebras

In quantum information, the algebra $\mathbb{C}\mathbb{Z}_m^{*n} \otimes \mathbb{C}\mathbb{Z}_m^{*n}$ models a physical system with two spatially separated subsystems, where in each subsystem we can make n different measurements, each with m outcomes. The recent MIP$^* =$ RE result shows that it is undecidable to determine whether an element of $\mathbb{C}\mathbb{Z}_m^{*n} \otimes \mathbb{C}\mathbb{Z}_m^{*n}$ (for varying n and m) is positive in all finite-dimensional representations. In this poster, I will present joint work with Arthur Mehta and William Slofstra, in which we show that it is undecidable to determine whether an element of $\mathbb{C}\mathbb{Z}_2^{*n} \otimes \mathbb{C}\mathbb{Z}_2^{*n}$ (for some sufficiently large n) is positive in all representations. As a consequence, there is no sum-of-squares certificate for positivity in tensor product of free algebras.

EUGENE ZIVKOV, Toronto Metropolitan University
Thin liquid film stability in the presence of bottom topography and surfactant

We consider the stability of gravity-driven fluid flow down a wavy inclined surface in the presence of surfactant. The periodicity of the bottom topography allows us to leverage Floquet theory to determine the correct form for the solution to the linearized governing partial differential equations. The result is that perturbations from steady state are wavelike, and a dispersion relation is identified which relates the wavenumber of an initial perturbation, κ, to its complex frequency, ω. The real part of ω ultimately determines the stability of the flow. We observe that the addition of surfactant generally has a stabilizing effect on the flow, but has a destabilizing effect for small wavenumbers. These results are compared and validated against nonlinear results, which are obtained by numerically solving the governing equations directly. The linear and nonlinear analyses show good agreement, except at small wavenumbers, where the linear results could not be replicated.