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Hadamard Product and Binomials Ideals

We study the Hadamard product of two varieties V and W, with particular attention to the situation when one or both of V and W is a binomial variety. The main result of this paper shows that when V and W are both binomial varieties, and the binomials that define V and W have the same binomial exponents, then the defining equations of $V \star W$ can be computed explicitly and directly from the defining equations of V and W. This result recovers known results about Hadamard products of binomial hypersurfaces and toric varieties. Moreover, as an application of our main result, we describe a relationship between the Hadamard product of the toric ideal I_G of a graph G and the toric ideal I_H of a subgraph H of G. We also derive results about algebraic invariants of Hadamard products: assuming V and W are binomial with the same exponents, we show that $\deg(V \star W) = \deg(V) = \deg(W)$ and $\dim(V \star W) = \dim(V) = \dim(W)$. Finally, given any (not necessarily binomial) projective variety V and a point $p \in \mathbb{P}^n \setminus \mathbb{V}(x_0 x_1 \cdots x_n)$, subject to some additional minor hypotheses, we find an explicit binomial variety that describes all the points q that satisfy $p \star V = q \star V$.