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Determining where Monte Carlo Outperforms Quasi-Monte Carlo for Functions Monotone in Each Coordinate in Dimensions 3 and Above

The Quasi-Monte Carlo methods are one way to estimate the integrals of functions over high-dimensional cubes. They are a variation of standard Monte Carlo methods; instead of choosing random points inside the cube to calculate an estimate of the result, Quasi-Monte Carlo scrambles a deterministic set of points that are sufficiently uniform inside of the cube. This is often desirable as it limits gaps and clusters of points that can harm the quality of the estimate.

One problem of interest to researchers of Quasi-Monte Carlo is to determine cases where these methods will outperform standard Monte Carlo methods, by having a lower theoretical variance in the final result. Previous work by Lemieux and Wiart showed that for two-dimensional functions monotone in each coordinate, Quasi-Monte Carlo will always outperform Monte Carlo in this way.

In this presentation, we will consider the extension of this problem to functions monotone in each coordinate in dimensions three and above. First, using computer searches we will find cases in higher dimensions where Monte Carlo has a lower theoretical variance than Quasi-Monte Carlo. Then, we will extend these cases to higher dimensions and determine relationships between them using equivalence classes and translations defined on sets of vectors called antichains.