WEIQI WANG, Concordia University

Compressive Fourier collocation methods for high-dimensional diffusion equations with periodic boundary conditions.

High-dimensional Partial Differential Equations (PDEs) are a popular mathematical modelling tool. However, standard numerical techniques for solving High-dimensional PDEs are typically affected by the curse of dimensionality. In this work, we tackle this challenge while focusing on stationary diffusion equations defined over a high-dimensional domain with periodic boundary conditions. Inspired by recent progress in high-dimensional sparse function approximation, we propose a new method called compressive Fourier collocation. Combining ideas from compressive sensing and spectral collocation, our method uses Monte Carlo sampling and employs sparse recovery techniques, such as orthogonal matching pursuit and l^1 minimization, to approximate the Fourier coefficients on given index sets of the PDE solution. We conduct a rigorous theoretical analysis showing that the approximation error of the proposed method is comparable with the best s-term approximation (with respect to the Fourier basis) to the solution and mitigates the curse of dimensionality with respect to the number of collocation points under sufficient conditions on the regularity of the diffusion coefficient. We present numerical experiments that illustrate the accuracy and stability of the method for the approximation of sparse and compressible solutions. In our current work, noticing that a bottleneck towards improving the solution accuracy is the choice of the index set, we develop a method using orthogonal matching pursuit to adaptively select the elements of the index set. In addition, we seek an efficient neural network model to solve the high-dimensional PDE, with the goal of comparing the performance of the adaptive method with a deep learning-based approach.