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Smooth approximations to the d-dimensional Haar system

In the late 1990s, Govil and Zalik showed how to approximate the system of Haar functions $h_{(I)}$ by smooth functions $\phi_{(I)}$, resulting in a system that was arbitrarily close to the Haar system in the sense of Bessel bounds. Later Zalik extended this result to d-dimensional Haar functions by taking tensor products. In 2001, Aimar, Bernardis, and Gorosito showed that the careful constructions of Govil and Zalik could be replaced (in one dimension) by convolutions with suitable smooth, even, mollifying functions. We show that "Zalik-like" approximations to the Haar system can be obtained in d dimensions by convolving the multidimensional Haar functions with essentially arbitrary compactly supported "mollifiers" that do not need to be smooth or have any special symmetry. These approximations to the Haar functions are stable (in the Bessel bound sense) with respect to small errors in dilation and translation and can be replaced by fine discretizations without producing too much additional error.