ALEX TUNG, University of Waterloo
Cheeger Inequalities for Vertex Expansion and Reweighted Eigenvalues

The classical Cheeger's inequality relates the edge conductance ϕ of a graph and the second smallest eigenvalue λ_2 of the Laplacian matrix. Recently, Olesker-Taylor and Zanetti discovered a Cheeger-type inequality $\psi^2/\log|V| \lesssim \lambda_2^* \lesssim \psi$ connecting the vertex expansion ψ of a graph $G=(V,E)$ and the maximum reweighted second smallest eigenvalue λ_2^* of the Laplacian matrix.

In this work, we first improve their result to $\psi^2/\log d \lesssim \lambda_2^* \lesssim \psi$ where d is the maximum degree in G, which is optimal up to constant factor. Also, the improved result holds for weighted vertex expansion, answering an open question by Olesker-Taylor and Zanetti. Building on this connection, we then develop a new spectral theory for vertex expansion. We discover that several interesting generalizations of Cheeger inequalities relating edge conductances and eigenvalues have a close analog in relating vertex expansions and reweighted eigenvalues. These include analogs of bipartite Cheeger's inequality, higher order Cheeger's inequality and improved Cheeger's inequality.

Finally, inspired by this connection, we present negative evidence to the $0/1$-polytope edge expansion conjecture by Mihail and Vazirani. We construct $0/1$-polytopes whose graphs have very poor vertex expansion. This implies that the fastest mixing time to the uniform distribution on the vertices of these $0/1$-polytopes is almost linear in the graph size. This does not disprove the conjecture, but this is in contrast with known positive results which proved poly-logarithmic mixing time to the uniform distribution on the vertices of subclasses of $0/1$-polytopes.

(Paper link: https://arxiv.org/abs/2203.06168.)