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Revisiting Degeneracy, Strict Feasibility, Stability in Linear Programming

Currently, the simplex method and the interior point method are indisputably the most popular algorithms for solving linear programs. Unlike general conic programs, linear programs, LPs, with a finite optimal value do not require strict feasibility in order to establish strong duality. Hence strict feasibility is seldom a concern, even though strict feasibility is equivalent to stability and a compact dual optimal set. This lack of concern is also true for other types of degeneracy of basic feasible solutions in LP. In this note we discuss that the specific degeneracy that arises from lack of strict feasibility necessarily causes difficulties in both simplex and interior point methods. In particular, we show that the lack of strict feasibility implies that every basic feasible solution, BFS, is degenerate; thus conversely, the existence of a nondegenerate BFS implies that strict feasibility (regularity) holds. We prove the results using facial reduction and simple linear algebra. In particular, the facially reduced system reveals the implicit non-surjectivity of the linear map of the equality constraint system. As a consequence, we emphasize that facial reduction involves two steps where, the first guarantees strict feasibility, and the second recovers full row rank of the constraint matrix. This illustrates the implicit singularity of problems where strict feasibility fails, and also helps in obtaining new efficient techniques for preproccessing. We include an efficient preprocessing method that can be performed as an extension of phase-I of the two-phase simplex method.