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Transient oscillations induced by delayed growth response in the chemostat

In order to try to account for the transient oscillations observed in chemostat experiments, we consider a model of single species growth in a chemostat that involves delayed growth response. The time delay models the lag involved in the nutrient conversion process.

By applying local and global Hopf bifurcation theorems, we prove that the model has unstable periodic solutions that bifurcate from unstable nonnegative equilibria as the parameter measuring the delay passes through certain critical values and that these local periodic solutions can persist, even if the delay parameter moves far from the critical (local) bifurcation values. When there are two positive equilibria, then positive periodic solutions can exist. When there is a unique positive equilibrium, the model does not have positive periodic oscillations and the unique positive equilibrium is globally asymptotically stable. However, the model can have periodic solutions that change sign. Although these solutions are not biologically meaningful, they may still help to account for the transient oscillations that have been frequently observed in chemostat experiments provided the initial data, though positive, starts close enough to the unstable manifold of one of these periodic solutions. Numerical simulations are provided to illustrate that the model has varying degrees of transient oscillatory behavior that can be controlled by the choice of the initial data.

This is joint work with Huaxing Xia and Lin Wang.