Set theory and its applications
Théorie des ensembles et ses applications
(Org: **Keegan Dasilva Barbosa** (Toronto) and/et **Paul Szeptycki** (York))

DAVOUD ABDI, University of Calgary
Counterexample to Conjectures of Bonato-Tardif, Thomassé and Tyomkyn

Two structures R and S are equimorphic when each embeds in the other; we may also say that one is a sibling of the other. Generally, it is not the case that equimorphic structures are necessarily isomorphic: the rational numbers, considered as a linear order, has up to isomorphism continuum many siblings. Let $Sib(R)$ be the number of isomorphism classes of siblings of R. Thomassé conjectured that for each countable relational structure R, $Sib(R) = 1$, $\aleph_0$ or $2^{\aleph_0}$. There is an alternative case of interest, whether $Sib(R) = 1$ or infinite for a relational structure R of any cardinality. The alternate Thomassé conjecture is connected to the Bonato-Tardif conjecture asserting that the sibling number of a tree is one or infinite. Further, Tyomkyn conjectured that if a locally finite tree has a non-surjective self-embedding, then it has infinitely many siblings, unless the tree is a ray. All the conjectures mentioned were verified for some structures such as chains, rayless trees, rooted trees, rayless graphs, cographs, countable NE-free posets, etc. This talk will introduce some locally finite trees having an arbitrary finite number of siblings, which disproves all conjectures of Bonato-Tardif, Thomassé and Tyomkin.

SHAUN ALLISON, University of Toronto
Polish groups involving S_∞

Say that a Polish group G involves a Polish group H iff there is a closed subgroup G_0 of G and a closed normal subgroup N of G_0 such that $G_0/N \cong H$. The group S_∞ is the Polish group of (full-support) permutations of $\mathbb{N}$. We show that the non-Archimedean Polish groups involving S_∞ has a deep and interesting theory, with several formulations that are equivalent. We use this theory to show that the non-Archimedean Polish groups which classify $=^+$ are exactly those which involve S_∞.

DANIEL CALDERON, University of Toronto
Borel's conjecture and meager-additive sets

Strong measure zero sets were introduced by Borel and have been studied since the beginning of the previous century. Borel conjectured that every strong measure zero set of real numbers must be countable. A few years later, Sierpiński proved that if the continuum hypothesis (CH) is assumed, then there exists an uncountable strong measure zero set of reals. Nevertheless, the question about the relative consistency of Borel's conjecture remained open until 1976 when Laver, in a ground-breaking paper, constructed a model of set theory in which every strong measure zero set of reals is countable.

A result of Galvin, Mycielski, and Solovay provides a characterization of Borel's strong nullity in terms of an algebraic (or translation-like) property for subsets of the real line. By means of this characterization, a strengthening of strong nullity, meager-additivity, appeared on the scene. Meager-additivity and other smallness notions on the real line have received considerable attention in recent years. A 1993 question due to Bartoszyński and Judah asks whether strong nullity and meager-additivity have a very rigid relationship, in the following sense:

Question (Bartoszyński–Judah, 1993): Suppose that every strong measure zero set of reals is meager-additive. Does Borel's conjecture follow?

We proved that it is relatively consistent with ZFC that every strong measure zero subset of the real line is meager-additive while there are uncountable strong measure zero sets (i.e., Borel's conjecture fails), giving a negative answer to the question above.

CESAR CORRAL, York University
Strong Fréchet properties, squares and AD families

We will deal with some questions about strengthenings of Fréchetness and the α_i properties introduced by Arhangel'skii. Using $\square(\kappa)$-sequences, we can build spaces which are Absolutely Fréchet and productively Fréchet but under some assumptions they may fail to be bisequential. We will also show that some of these properties can be obtained in ZFC using almost disjoint families.

TETSUYA ISHIU, Miami University
The Mardešić Conjecture for Countably Compact Spaces

We shall outline the proof that for all positive integers d and s, if Z_j is an infinite Hausdorff space for each $j < d+s$ and $\prod_{j<d+s} Z_j$ is a continuous image of a countably compact subspace of the product of d-many compact linearly ordered topological spaces, then there are at least $s+1$-many indexes $j < d+s$ such that Z_j is compact and metrizable. This theorem is a strengthening of the Mardešić Conjecture, which was proved by G. Martínez-Cervantes and G. Plebanek in 2019, but it was proved by a completely different method.

SUMUN IYER, Cornell University
Dynamics of the Knaster continuum homeomorphism group

We use the projective Fraissé approach and Ramsey's theorem to show that the universal minimal flow of the homeomorphism group of the universal Knaster continuum is homeomorphic to the universal minimal flow of the free abelian group on countably many generators.

Knaster's continuum is a compact, connected metrizable space which is indecomposable: in the sense that it is not the union of two non-trivial compact, connected, metrizable subsets. We will define a projective Fraissé class whose limit approximates the universal Knaster continuum in such a way that the group $\mathrm{Aut}(\mathbb{K})$ of automorphisms of the Fraissé limit is a dense subgroup of the group, $\mathrm{Homeo}(K)$, of homeomorphisms of the universal Knaster continuum. The computation of the universal minimal flow involves modifying the Fraissé class in a natural way so that it approximates an open, normal, extremely amenable subgroup of $\mathrm{Homeo}(K)$.

VINICIUS RODRIGUES, York University
Special sets of reals and weakenings of normality in Isbell-Mrówka spaces

The problem of the existence of a non-metrizable separable normal Moore space is a classical problem related to uncountable normal Isbell-Mrówka spaces and with the class of Q-sets - which are special subsets of the reals: the existence of these objects are independent of ZFC and equivalent. Using the same techniques as the ones needed to prove this equivalence, analogous relations between uncountable pseudonormal Isbell-Mrówka spaces and λ-sets were established.

Taking as a motivation, we will discuss the relations between some weakenings of normality in Isbell-Mrórka spaces (as almost-normality and strong $\aleph_0$-separatedness) and other special subsets of the reals (such as σ-sets and a new class of subsets of the reals we are proposing, which we called weak λ-set). In particular, we prove that a branching almost disjoint family generated by a set of reals is almost-normal (strong $\aleph_0$-separated) if, and only if the set of reals is a σ-set (weak λ-set). This is a joint work with V. S. Ronchim and P. Szeptycki.

FRANKLIN TALL, University of Toronto
An undecidable extension of Morley's theorem on the number of countable models

AN UNDECIDABLE EXTENSION OF MORLEY"S THEOREM ON THE NUMBER OF COUNTABLE MODELS

Franklin D. Tall

This is joint work with Christopher J. Eagle, Clovis Hamel, and Sandra Muller. We show that Morley's theorem on the number of countable models of a countable first-order theory becomes an undecidable statement when extended to second-order logic. More generally, we calculate the number of equivalence classes of sigma-projective equivalence relations in several models of set theory. Our methods include random and Cohen forcing, Woodin cardinals and Inner Model Theory.

JING ZHANG, University of Toronto