Representation Theory of Algebras
Théorie de la représentation des algèbres
(Org: , **Kaveh Mousavand** (Queen's) and/et **Hugh Thomas** (UQAM))

EMILY BARNARD, DePaul University
Triangulations and maximal almost rigid representations

Let kQ/I be a finite representation type gentle algebra. Two modules M and N are called almost rigid if they do not have any nonsplit extensions or if any extension between M and N is indecomposable. A module T is maximal almost rigid (mar) if its indecomposable summands form a maximal collection of pairwise almost rigid modules. In this talk, we show each mar module T has the same number of summands. We use a modified version of the surface model for the modules of kQ/I developed by Coelho-Simoes and Baur to show that each mar T corresponds bijectively to a permissible triangulation of our surface. Finally, we show that the endomorphism algebra of a mar module over kQ/I is the endomorphism algebra of a tilting module over a bigger gentle algebra. Our results generalize the hereditary type A case, where the mar modules correspond to triangulations of a polygon, and their endomorphism algebras are tilted algebras.

THOMAS BRÜSTLE, Bishop's University and Université de Sherbrooke
Relative torsion classes

We study the notion of torsion classes relative to exact structures. This is motivated by the notion of almost rigid objects introduced by Emily Barnard, Emily Gunawan, Emily Meehan and RalfSchiffler. More generally, the aim is to study cluster-like structures for exact categories.

WILL DANA, University of Michigan
Walls of shards and filtrations of shard modules

The representation theory of preprojective algebras is intertwined with the combinatorics of Coxeter groups. In particular, for a simply laced Dynkin diagram, the King stability domains of bricks of its preprojective algebra partition the reflecting hyperplanes of the associated Coxeter group into cones called shards, which previously arose in work of Nathan Reading on the lattice structure of the weak order. In joint work with David Speyer and Hugh Thomas, we extend this result to non-Dynkin diagrams by showing a bijection between rigid bricks with full-dimensional stability domain (which we call "shard modules") and shards.

In this talk, we'll give an overview of this generalization, and then showcase a couple of ways in which the geometry of shards relates to properties of shard modules. We'll generalize a result from the Dynkin case, showing that, at any wall of a shard, it's met by two other shards such that the three corresponding shard modules fit into a nice short exact sequence. We'll then apply this result to families of diagrams with "tails" to visualize how shard modules are related to the positions of their shards in this case.

BENJAMIN DEQUÊNE, Université du Québec à Montréal
Some Jordan recoverable subcategories of modules over gentle algebras.

Gentle algebras form a class of finite dimensional algebras introduced by Assem and Skowronski in the '80s. Modules over such an algebra can be described by string and band combinatorics, which are some kind of walk in the associated gentle quiver, thanks to the works of Butler and Ringel. A subcategory $\mathcal{C}$ of modules is said to be Jordan recoverable if we can recover (up to isomorphism) a representation X in $\mathcal{C}$ from the Jordan form of its generic nilpotent endomorphisms, called the generic Jordan form data of X.

The main aim of the talk is to explain the notion of Jordan recoverability through various examples and to highlight a combinatorial characterization of that property for some special kind of subcategories of modules over gentle algebras - a result which extends the work of Garver, Patrias, and Thomas done in Dynkin cases. If time allows it, we could discuss some open questions related to this result.

This is a part of my Ph.D. work supervised by Hugh Thomas.

COLIN INGALLS, Carleton University
Sets of mutually orthogoval projective and affine planes

A pair of planes, both projective or both affine, of the same order and on the same pointset are orthogoval if each line of one plane intersects each line of the other plane in at most two points. In this paper we prove new constructions for sets of mutually orthogoval planes, both projective and affine, and review known results that are equivalent to sets of more than two mutually orthogoval planes. We also discuss the connection between sets of mutually orthogoval planes and covering arrays. This is joint work with Charles J. Colbourn, Jonathan Jedwab, Mark Saaltink, Ken W. Smith, and Brett Stevens.

SHIPING LIU, Université de Sherbrooke
Module categories with a null forth power of the radical

This is a joint work with Youqi Yin. Motivated by the well known fact that an artin algebra is semi-simple if and only if its module category has a null radical, our work aims to initiate a study of the representation theory of representation-finite artin algebras in terms of the nilpotency of the radical of their module category. In this talk, we shall give a complete list of artin algebras A such that $\mathrm{rad}^4(\mathrm{mod}\, A) = 0$.

CHARLES PAQUETTE, Royal Military College of Canada
Biserial algebras and bricks

Biserial algebras form an important class of tame algebras, which include the class of special biserial algebras. In this talk, we study the bricks over such algebras, which are the modules having a division algebra as endomorphism algebra. For a general finite dimensional algebra A, bricks over A form an important family of indecomposable modules. They are the simple objects of the wide subcategories of the module category, they include all stable modules in the sense of King, and more recently, have shown to be deeply connected to the τ-tilting theory and the so-called wall-and-chamber structure of A. We will show that a biserial algebra B has finitely many bricks (then called brick-finite) if and only if no quotient of B is gentle of infinite type. The latter can be detected by classifying the minimal brick-infinite biserial algebras, that is, the biserial algebras which are brick-infinite but such that every proper quotient is brick-finite. We also consider some infinite dimensional bricks (called generic bricks) and explain how the existence of those is equivalent to the existence of infinite families of bricks. This is joint work with Kaveh Mousavand.

DEEPANSHU PRASAD, Queen's University
An Extension of Sato-Kimura Theorem for Semi-invariant rings

We give an analog of a result of Sato-Kimura in the context of a semi-invariant ring for a finite dimensional algebra over an algebraically closed field $\mathbb{K}$ of characteristics 0. Then, we examine the case of finite dimensional hereditary algebras over $\mathbb{K}$ and give another proof of a result by Paquette and Weyman. This is joint work with Charles Paquette and David Wehlau.

YVAN SAINT-AUBIN, Université de Montréal
Spin chains as modules over the affine Temperley-Lieb algebra

Let $V = (\mathbb{C}^2)^{\otimes N}$ be the tensor product of N copies of the two-dimensional simple $U_q(sl_2)$-module. It is also a $U_q(sl_2)$-module (through the coproduct on $U_q(sl_2)$). The algebra of endomorphisms $\mathrm{End}_{U_q(sl_2)}V$ is known to define a representation of the (original) Temperley-Lieb algebra TL_N on V (Jimbo (1985, 1986), Martin (1992)). This is known as the (q-)Schur-Weyl duality. The TL_N-action on V was extended to one of the affine Temperley-Lieb algebra aTL_N by two physicists in their study of spin chains (Pasquier and Saleur (1990)). While this extended action fails to commute with that of $U_q(sl_2)$, the interplay between both actions can be used to reveal the structure of V as a aTL_N-module.

This is joint work with Théo Pinet (arXiv:2205.02649).

CHARLES SENÉCAL, Université de Montréal

Centralizers of products of $LU_q(\mathfrak{sl}_2)$-modules at roots of unity

Let V be the fundamental representation of the quantum group $U_q(\mathfrak{sl}_2)$. Quantum Schur-Weyl duality says that the centralizer of the action of $U_q(\mathfrak{sl}_2)$ on the product $V^{\otimes n}$ is isomorphic to the Temperley-Lieb algebra $\mathsf{TL}_n(q+q^{-1})$, even when q is a root of unity (in which case we consider the action of Lusztig's extension $LU_q(\mathfrak{sl}_2)$). We explore products other than $V^{\otimes n}$, namely we describe the centralizer of the action of $LU_q(\mathfrak{sl}_2)$ on $P \otimes V^{\otimes n}$, where P is a projective $LU_q(\mathfrak{sl}_2)$-module. We give a complete description of the algebra $\mathsf{End}_{LU_q(\mathfrak{sl}_2)}(P \otimes V^{\otimes n})$ in both cases when q is a root of unity or not. This is joint work with Yvan Saint-Aubin.

KHRYSTYNA SERHIYENKO, University of Kentucky

Title: Leclerc's conjecture on a cluster structure for type A Richardson varieties

Leclerc constructed a conjectural cluster structure on Richardson varieties in simply laced types using cluster categories coming from preprojective algebras. We show that in type A, his conjectural cluster structure is in fact a cluster structure. We do this by comparing Leclerc's construction with another cluster structure due to Ingermanson, which uses the combinatorics of wiring diagrams and the Deodhar stratification. Though the two cluster structures are defined very differently, we show that the quivers coincide and clusters are related by the twist map for Richardson varieties, recently defined by Galashin–Lam. This is join work with Melissa Sherman-Bennett.

DAVID SPEYER, University of Michigan

Coxeter groups and torsion classes of quiver and preprojective algebras

I'll describe the relationship between the combinatorics of Coxeter groups, and the representation theory of quiver path algebras and preprojective algebras. I'll start back with Colin Ingalls and Hugh Thomas's 2006 paper "Noncrossing partitions and representations of quivers", explain how Nathan Reading and my work on Cambrian lattices fits in, and describe work in progress with Hugh Thomas and Will Dana.

GORDANA TODOROV, Northeastern University

Nakayama Algebras which are Defect Invariant

(Joint work: Emre Sen, Shijie Zhu, GT)

It was very beautiful idea of Emre Sen to consider "syzygy filtrations" and create epsilon construction which he used to prove several homological statements about Nakayama algebras: about φ-dimension, Gorenstein dimension, finitistic dimension. Now, in this joint work we consider the process of reversing epsilon construction, while preserving the defect of the algebras; we apply this to give complete classification of Nakayama algebras which are Auslander-Gorenstein and finitistic Auslander algebras.

NICHOLAS WILLIAMS, Lancaster University

Cyclic polytopes and representation theory

Oppermann and Thomas show how the representation theory of Iyama's higher Auslander algebras of type A (A_n^d) is related to triangulations of even-dimensional cyclic polytopes. We show how two natural partial orders on the set of triangulations of a cyclic polytope, the higher Stasheff–Tamari orders, can be interpreted on the representation-theoretic side as well-known orders on silting complexes introduced by Aihara and Iyama. This allows one to interpret triangulations of odd-dimensional cyclic polytopes within the representation theory of A_n^d, namely, as equivalence classes of d-maximal green sequences. This allows the higher Stasheff–Tamari orders to be interpreted algebraically in odd dimensions too. Finally, we prove the 1996 conjecture of Edelman and Reiner that the two higher Stasheff–Tamari orders are equal, and thereby obtain new results on the representation theory of A_n^d.

MILEN YAKIMOV, Northeastern University
Poisson geometry and representation theory of root of unity quantum cluster algebras

We will show that all root of unity quantum cluster algebras have canonical structures of Cayley-Hamilton algebras (in the sense of Procesi) and Poisson orders (in the sense of De Concini-Kac-Procesi and Brown-Gordon). The first result allows to transfer finiteness properties between the quantum and classical situations. The second result relates the representation theory of these algebras to the Poisson geometry of the Gekhtman-Shapiro-Vainshtein brackets. We will then prove that the spectrum of each upper cluster algebra equipped with the GSV Poisson structures has an explicit Zariski open torus orbit of symplectic leaves, which is a far reaching generalization of the Richardson divisor of a Schubert cell in Lie theory. At the end we will combine the above results to describe explicitly the fully Azumaya loci of the root of unity quantum cluster algebras. This classifies their irreducible representations of maximal dimension. This is a joint work with Shengnan Huang, Thang Le, Greg Muller, Bach Nguyen and Kurt Trampel.