
Matrix Analysis and Operator Theory (Bilingual Session)

Analyse matricielle et théorie des opérateurs

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LUDOVICK BOUTHAT, Université Laval

The convergence of Doubly Stochastic Markov Chains

In recent years, some interest has been devoted to studying doubly stochastic Markov chains. These chains appears naturally in many real-life applications such as quantum measurements. In this note, we proceed to characterize the asymptotic behavior of an homogeneous doubly stochastic Markov chains. In particular, we characterize the doubly stochastic matrices whose associated Markov chain (1) describes a cycle; (2) converges to a given matrix; and (3) diverges. We also provide a new sufficient condition for the infinite product of doubly stochastic matrices $A_1 A_2 A_3 \cdots$ to converge to a scalar multiple of the all-ones matrix, thus improving a result of Schwarz.

GALIA DAFNI, Concordia University

Approximate moment conditions for h^p atoms and molecules, and the boundedness of inhomogeneous Calderón–Zygmund operators

In joint work with Chun Ho Lau (Concordia), Tiago Picon (Universidade São Paulo), and Claudio Vasconcelos (Universidade Federal de São Carlos), we show that atoms and molecules for the local Hardy spaces $h^p(\mathbb{R}^n)$, $0 < p \leq 1$, must satisfy certain logarithmic-type moment conditions. From this one can obtain the necessity of the approximate cancellation conditions we have previously imposed in order to show the boundedness of inhomogeneous Calderón–Zygmund operators on these spaces.

DAVID FEDER, Calgary

The matrix permanent and determinant as an eigenvalue problem

The permanent of a square matrix is the symmetric analogue of the usual determinant, but where the signatures of the permutations (i.e. the signs appearing in the expansion of the function) are ignored. Despite the fact that both of these functions yield the same exponential number of terms, the determinant is efficiently computable classically; in contrast, determining the permanent of a matrix is $\#P$ -hard, and the discovery of a classically efficient algorithm would have profound consequences for the theory of computation and for recent claims of quantum supremacy. I will introduce an approach to computing the determinant and permanent, via the spectrum of the adjacency matrix of a weighted directed hypercube graph. Gaussian elimination of the original matrix corresponds to deleting vertices and reweighting edges of the adjacency matrix, projecting out the generalized zero eigenvectors while preserving the non-zero eigenvalues. I will discuss how the determinant and permanent respectively map to non-interacting spinless fermions and hard-core bosons hopping on a one-dimensional lattice, and approaches to obtaining the permanent via a quantum algorithm. This is collaborative work with Abhijeet Alase

DAVID KRIBS, University of Guelph

Graph theory, matrix theory, and operator theory, and distinguishing quantum states via LOCC

In this talk, I'll discuss my ongoing work with collaborators on a basic topic in quantum communication theory: Given a set of known quantum states, when can two parties distinguish the states via the hybrid classical-quantum communication protocol called local (quantum) operations and classical communication (LOCC). I'll focus on the case of quantum product states, and will show how we've been able to make use of aspects of graph theory, matrix theory, and operator theory to develop techniques for distinguishing such states in the one-way LOCC framework. This talk is based on joint works with Comfort Mintah, Michael Nathanson, and Rajesh Pereira.

MILIVOJE LUKIC, Rice University
Stahl–Totik regularity for Schrodinger operators

This talk describes a theory of regularity for one-dimensional continuum Schrodinger operators. For any half-line Schrodinger operator with a bounded potential V , we obtain universal thickness statements for the essential spectrum, in the language of potential theory and Martin functions (which will be defined in the talk). Namely, we prove that the essential spectrum is not polar, it obeys the Akhiezer–Levin condition, and moreover, the Martin function at infinity obeys the two-term asymptotic expansion $\sqrt{-z} + \frac{a}{2\sqrt{-z}} + o(\frac{1}{\sqrt{-z}})$ as $z \rightarrow -\infty$. The constant a in its asymptotic expansion plays the role of a renormalized Robin constant and enters a universal inequality $a \leq \liminf_{x \rightarrow \infty} \frac{1}{x} \int_0^x V(t) dt$. This leads to a notion of regularity, with connections to the exponential growth rate of Dirichlet solutions and limiting eigenvalue distributions for finite restrictions of the operator, and applications to decaying and ergodic potentials. This is joint work with Benjamin Eichinger.

FRÉDÉRIC MORNEAU-GUÉRIN, TÉLUQ
Sur une question posée par Erdős au sujet des matrices doublement stochastiques

Dans un célèbre article de Marvis Marcus et Rimhak Ree datant de 1958, on démontre que pour toute matrice doublement stochastique A on a que le carré de la norme de Frobenius de A est inférieur est borné par la trace d'une diagonale de A . Paul Erdős a ensuite posé la question suivante : sous quelle(s) condition(s) cette inégalité est-elle stricte ?

Au cours de cette présentation, nous présenterons une réponse partielle de Marcus et Ree s'appliquant dans le cas général ainsi qu'une réponse explicite, complète et détaillée s'appliquant dans un cas particulier.

MARCU-ANTONE ORSONI, University of Toronto
Separation of singularities for the Bergman space and reachable space of the heat equation.

Let Ω_1 and Ω_2 be two open sets of the complex plane with non empty intersection. The separation of singularities problem can be stated as follows: if f belongs to the Bergman space of $\Omega_1 \cap \Omega_2$, can we find f_1 and f_2 belonging respectively to the Bergman spaces of Ω_1 and Ω_2 , such that $f = f_1 + f_2$? In this talk, we will see general settings in which the previous question has a positive answer and we will apply these results to the description of the reachable space of the heat equation. Joint work with Andreas Hartmann.

PIERRE-OLIVIER PARISÉ, University of Hawaii at Manoa
Infinite Matrices of Operators

A summability method can be given as an infinite matrix of the form

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & \cdots \\ a_{1,0} & a_{1,1} & a_{1,2} & \cdots \\ a_{2,0} & a_{2,1} & a_{2,2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad a_{i,j} \in \mathbb{C}.$$

In this talk, based on A. Robinson's article *On Functional Transformation and Summability*, I will present a generalization of a summability method where each entry $a_{i,j}$ is replaced by a bounded linear operator on a Banach space. I will also show a generalization of the Silverman-Toeplitz Theorem in this new framework.

Note: I will present in English, but the slides will be in French.

IGNACIO URIARTE-TUERO, University of Toronto
An operator theoretic application of two weight norm inequalities for SIOs in \mathbb{R}^n

I will report on recent progress on the two weight problem for singular and fractional integral operators in \mathbb{R}^n , in particular a new stability result for boundedness of certain operators (joint with M. Alexis, J.L. Luna Garcia, and E. Sawyer). The talk will be self-contained.