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Deep Reinforcement Learning for Viscous Incompressible Flow

Numerical methods for approximating the solution to the incompressible Navier-Stokes equations typically solve discretized equations on a finite mesh of the domain, a computationally expensive process. We present a mesh-free method which can be easily scaled to irregular 3D geometries as we encode the domain and boundary through signed distance functions. The numerical solution is provided by a deep neural network trained on an objective that is derived from the expectation of a martingale stochastic process of the viscous Burgers equation, similar to Monte Carlo methods through the Feynman-Kac formula. We adopt a reinforcement learning paradigm of iterating the optimization step at every simulated increment of the Itô process. The vector potential is encoded into the neural network architecture, thereby automatically satisfying the incompressibility condition without requiring the pressure term. Simulation of the Itô process requires the true velocity, which we replace with the current approximation during the training procedure and we prove that this process is a fixed-point iteration in a simplified setting. This method is capable of numerically solving solutions to elliptic and parabolic partial differential equations. Deep learning is parallelizable and hyperparameters can be incorporated to solve a family of problems. We provide an example of flow past disk with a range of input flow speeds and viscosities, all provided by a single neural network, to highlight these advantages.