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On the Best Way to Play with Fire: an Adversarial Burning Game

Graph burning is a discrete-time process that models the spread of influence in a network. Vertices are in one of two states: either burning or unburned. In each round, a burning vertex causes all of its neighbours to become burning and then a previously unburned vertex is chosen whose state is changed to burning. Previous work has focused on bounding the number of turns necessary to burn an n-vertex graph. We introduce a variation of the graph burning process that incorporates an adversarial game played on a nested, growing sequence of trees. Two players, Arsonist and Builder, play in turns: Builder adds unburned vertices to create a larger tree, then burning vertices spread fire to their neighbours, and finally Arsonist 'lights' a new fire on an unburned vertex. This process repeats forever. Arsonist is said to win if the limiting fraction of burned vertices tends to 1, while the Builder is said to win if this fraction is bounded away from 1. We consider how the number of vertices granted to Builder each turn affects the optimal strategies for each player.