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Degenerate Ellipticity and Hypoellipticity for Divergence Operators

We discuss differential operators of the form $L = -\mathrm{div}(Q\nabla\cdot)$, where Q is a non-negative definite symmetric matrix-valued function. Even though L fails to be elliptic at points where Q is singular, in some settings we can recover regularity results for weak solutions to the Dirichlet problem

$$\begin{aligned} Lu &= f \quad x \in \Omega, \\ u &= 0 \quad x \in \partial\Omega. \end{aligned}$$

In this talk, I will present some recent joint work with S. Rodney concerning sufficient conditions for a priori boundedness of weak solutions to these problems. We show that even if a very weak Sobolev inequality with gain on Orlicz scale holds, one can still recover boundedness under appropriate hypotheses on the data f.

On the other hand, if rapid degeneracy of Q forbids a Sobolev inequality with gain on Orlicz scale from holding, it would still be useful to understand qualitative behaviours of the operator L. To this end, I also discuss recent work at finding sufficient conditions for hypoellipticity of second-order divergence operators by using decompositions of Holder continuous functions into sums of squares. This work complements some recent results by Sawyer and Korobenko, which show that $L = -\mathrm{div}(Q\nabla\cdot)$ is hypoelliptic when the matrix Q admits a suitable decomposition.