MICHEL DELFOUR, Université de Montréal
Control, Shape, and Topological Derivatives via Minimax Differentiability of Lagrangians

In Control Theory, the semidifferential (a one-sided directional derivative) of a state constrained objective function can be obtained by introducing a Lagrangian and an adjoint state. This problem is equivalent to the one-sided derivative of the minimax of the parametrized Lagrangian with respect to a positive parameter t as it goes to 0 (for instance, Delfour and Zolésio, Shape and Geometries, Metrics, Analysis, Differential Calculus and Optimization, SIAM Ser. Advance in Control and Design, 2011] and Sturm, SIAM J. on Control and Optim., 53, no. 4, 2017-2039]. In this talk new simpler conditions that predict the occurrence of an extra term (the polarization term in Mechanics) are given in term of the standard adjoint [Delfour, Control, shape, and topological derivatives via minimax differentiability of Lagrangians, Springer INdAM Series Vol. 29, 2018]. They are applied to the computation of semidifferentials with respect to the control and the shape and the topology of the underlying domain [Delfour, Topological Derivative of State Constrained Objective Functions: a Direct Approach, SIAM J. on Control and Optim. (1) 60 (2022), 22-47]. The shape derivative is a differential while the topological derivative usually obtained by expansion methods is not. It is a semidifferential obtained by perturbations arising from dilatations of points, curves, surfaces and, potentially, microstructures by using the notion of d-dimensional Minkowski content. Examples of such perturbations are the d-rectifiable sets and the sets of positive reach of Federer [Delfour, Topological derivatives via one-sided derivative of parametrized minima and minimax, Engineering Computations (1) 39 (2022), 34-59].