Approximation Theory, Function Spaces and Harmonic Analysis
Théorie de l'approximation, espaces de fonctions et analyse harmonique
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ALMUT BURCHARD, University of Toronto
Strong and weak maximal extensions of $\overline{\partial}$ on the Hartogs triangle

The Hartogs triangle $\mathbf{T} := \{(z,w) \in \mathbb{C}^2 : |z| < |w| < 1\}$ has been an important source of (counter-) examples in several complex variables. I will discuss recent work with J. Flynn, G. Lu, and M.-C. Shaw on properties of the Sobolev space H^1 and the maximal extensions of the Cauchy-Riemann operator $\overline{\partial}$ on $\mathbf{T}$. It turns out that despite the singularity at the origin, $\mathbf{T}$ shares many properties of Lipschitz domains.

FENG DAI, University of Alberta
Marcinkiewicz-type discretization for functions from a finite dimensional space

In this talk I will report some recent results on Marcinkiewicz-type discretization of integral norms for functions from finite dimensional linear spaces. In particular, I will describe the recent progress made by A. Prymak and myself on optimal polynomial meshes on convex bodies.

OSCAR DOMINGUEZ, Universidad Complutense Madrid, CRM-Montreal
Truncated smooth function spaces

We introduce truncated smooth function spaces. These spaces have many interesting properties and are useful to solve several outstanding questions in functional analysis and PDE's. The talk is based on joint work with Sergey Tikhonov.

JOSHUA FLYNN, CRM-ISM, McGill University
Helgason-Fourier Analysis and Sharp Geometric Inequalities on the Rank One Symmetric Spaces

The Hardy-Sobolev-Maz'ya inequality combines the Hardy and Sobolev inequalities into a single inequality on the halfspace. Using conformal equivalence, this inequality is equivalent to the Poincare-Sobolev inequality on the real hyperbolic space. Using Helgason-Fourier analysis, higher order versions of these inequalities were established by G. Lu and Q. Yang for the real and complex hyperbolic spaces. With G. Lu and Q. Yang, we further established these inequalities for the quaternionic and octonionic hyperbolic spaces. In this talk we will present these results and the Fourier analytic tools used in obtaining them.

RYAN GIBARA, University of Cincinnati
A Dirichlet problem for unbounded domains in metric measure spaces

Let Ω be an unbounded locally compact metric measure space that is uniform in its completion $\overline{\Omega}$. When Ω is equipped with a doubling measure satisfying a p-Poincaré inequality and the boundary $\partial\Omega := \overline{\Omega} \setminus \Omega$ is bounded, we solve the p-Dirichlet problem for boundary data in an appropriate Besov class.

This is accomplished by transforming both the metric and the measure on Ω using a weight that depends on the distance to the boundary, rendering Ω bounded while retaining many of its metric and measure properties without perturbing the space near the boundary.

This is joint work with Rikka Korte and Nageswari Shanmugalingam.

BIN HAN, University of Alberta
Gibbs Phenomenon of Wavelets and Quasi-projection Approximation

Most data such as images are piecewise smooth functions. It is well known that the standard Fourier series approximation suffers the unpleasant ringing effect near discontinuity, which is termed as the Gibbs phenomenon such that the nth Fourier partial sums overshoot a function at jump discontinuities and the overshoot does not die out as n goes to infinity. Wavelets and framelets are known to be the mainstream multiscale sparse representation and approximation systems in data science. In this talk we study the Gibbs phenomenon of framelet/wavelet expansions and their associated quasi-projection approximation schemes at an arbitrary point. We show that the Gibbs phenomenon appears at all points for every tight or dual framelet having at least two vanishing moments and for quasi-projection approximation operators having at least three accuracy/approximation orders. This well explains the ringing effect of most wavelet approximation in applications. We shall also address how to avoid the Gibbs phenomenon for wavelets/framelets and quasi-projection approximation, as well as address the Gibbs phenomenon for approximation through sampling. This talk is based on [B. Han, Gibbs phenomenon of framelet expansions and quasi-projection approximation, Journal of Fourier Analysis and Applications, 25 (2019), 2923-2956].

ALEX IOSEVICH, University of Rochester
Frame theory and finite point configurations

We are going to discuss a series of intriguing connections between the existence and non-existence of exponential frames and point configuration problems in geometric measure theory and combinatorics.

CINTIA PACCHIANO, University of Calgary
Existence of parabolic minimizers to the total variation flow on metric measure spaces

In this project, we discuss some fine properties and the existence of variational solutions to the Total Variation Flow. Instead of the classical Euclidean setting, we intend to work mostly in the general setting of metric measure spaces. During the past two decades, a theory of Sobolev functions and BV functions has been developed in this abstract setting. A central motivation for developing such a theory has been the desire to unify the assumptions and methods employed in various specific spaces, such as weighted Euclidean spaces, Riemannian manifolds, Heisenberg groups, graphs, etc.

The total variation Flow can be understood as the process of diminishing the total variation using the gradient descent method. This idea can be reformulated using parabolic minimizers, and it gives rise to a definition of variational solutions. The approach's advantages using a minimization formulation include much better convergence and stability properties. This is essential as the solutions naturally lie only in the space of BV functions.

We give an existence proof for variational solutions u associated to the total variation flow. Here, the functions being considered are defined on a metric measure space (X, d, μ). For such parabolic minimizers that coincide with a time-independent Cauchy-Dirichlet datum u_0 on the parabolic boundary of a spacetime-cylinder $\Omega \times (0, T)$ with $\Omega \subset X$ an open set and $T > 0$, we prove the existence in the weak parabolic function space $L^1_w(0, T; BV(\Omega))$. This is a joint project with Vito Buffa and Michael Collins.

THOMAS RANSFORD, Université Laval
Constructive polynomial approximation

Let X be a function space. Here are two problems about polynomial approximation in X: (1) Are polynomials dense in X? (2) If so, then, given $f \in X$, can we find explicit polynomials p_n that converge to f in X? In this talk, I shall discuss some recent work on the second type of problem. (Joint with Javad Mashreghi and Pierre-Olivier Parisé).

SCOTT RODNEY, Cape Breton University
More Limits of Orlicz Norms

In this talk I will discuss recent work with my recent honours student A. Mailhot, where we recover $\|f\|_\infty$ as a limit of Orlicz norms of f defined by a one parameter family of iterated log-bump type Young functions. I will also put this work into context with recent advances with S. F. MacDonald.

MICHAEL ROYSDON, ICERM, Brown University and CRM, Concordia University
Weighted Projection Bodies

The inequalities of Petty and Zhang are affine isoperimetric inequalities, the former of which implies that classical isoperimetric inequality and is equivalent to an affine version of the Sobolev inequality for compactly support C^1 functions, while the latter is a very strong reverse isoperimetric inequality. Each of these inequalities feature a certain class of convex bodies, called projection bodies, which may be described in terms of the cosine transform of the surface area measure of a given convex body.

In this talk, we will discuss a generalization of these bodies to the weighted setting (by replacing the surface area measure with different measures satisfying mild regularity conditions) and describe how they may be used to prove strong reverse isoperimetric inequalities. And, in addition, show how these results may be used to imply a reverse form of the isoperimetric inequality for certain classes of measures on the n-dimensional Euclidean space.

This is based on a joint work with D. Langharst and A. Zvavitch.

ALEJANDRO SANTACRUZ-HIDALGO, Western University
Down spaces over a measure space with an ordered core

We consider a measure space together with a totally ordered subset of its sigma algebra called an *ordered core*. Recently, this construction was used in the context of Hardy inequalities, giving a uniform treatment of many different types of Hardy operators.

We will begin by introducing a definition of monotone functions compatible with the ordered core. This allows us to extend the down space construction, a variant of the Köthe dual restricted to positive decreasing functions, to all measure spaces. We will look at their associate spaces and their relationship with a suitable version of the least decreasing majorant construction in this more general setting. We will discuss the interpolation structure of these spaces and find strong similarities to the real line case; the down spaces corresponding to L^1 and L^∞ form an exact Calderón-Mityagin couple and as a consequence we can describe all their exact interpolation spaces in terms of the $K-$functional. We will also show an analogous result for the dual couple.

This talk is based on joint work with Gord Sinnamon.

ERIC SAWYER, McMaster University
Two weight T1 theorems for Sobolev and Lp spaces with doubling measures and Calderón-Zygmund operators.

This is joint work with Brett Wick. We characterize two weight norm inequalities for Calderón-Zygmund operators from one weighted space to another, when the measures are doubling. We extend an earlier result of Michel Alexis, the speaker and Ignacio Uriarte-Tuero for L2 spaces, to L2-Sobolev spaces of small order, and to Lp spaces. In the case p is not 2, we use variants of the quadratic Muckenhoupt conditions and weak boundedness properties introduced by Hytönen and Vuorinen. In particular, this proves their conjecture for the Hilbert transform in the case of doubling measures.

GORD SINNAMON, Western University, London, Canada
The Fourier transform in rearrangement-invariant spaces

We will look at the Fourier transform as a map between rearrangement-invariant spaces of functions on $\mathbb{R}^n$. Restricting our attention to domain spaces that are 2-concave and range spaces that are 2-convex, we give necessary and sufficient conditions for boundedness.

IGNACIO URIARTE-TUERO, University of Toronto
Two weight norm inequalities for singular and fractional integral operators in $\mathbb{R}^n$

I will report on recent progress on the two weight problem for singular and fractional integral operators in $\mathbb{R}^n$, in particular a local Tb theorem in $\mathbb{R}^n$ for general measures with an energy side condition (joint with C. Grigoriadis, M. Paparizos, E. Sawyer and C.-Y. Shen) and a two weight T1 theorem (with no side conditions) for doubling measures (joint with M. Alexis and E. Sawyer), and briefly mention a new stability result. The talk will be self-contained.

MICHAEL WILSON, University of Vermont
Smooth approximations to the d-dimensional Haar system

In the late 1990s, Govil and Zalik showed how to approximate the system of Haar functions $h_{(I)}$ by smooth functions $\phi_{(I)}$, resulting in a system that was arbitrarily close to the Haar system in the sense of Bessel bounds. Later Zalik extended this result to d-dimensional Haar functions by taking tensor products. In 2001, Aimar, Bernardis, and Gorosito showed that the careful constructions of Govil and Zalik could be replaced (in one dimension) by convolutions with suitable smooth, even, mollifying functions. We show that "Zalik-like" approximations to the Haar system can be obtained in d dimensions by convolving the multidimensional Haar functions with essentially arbitrary compactly supported "mollifiers" that do not need to be smooth or have any special symmetry. These approximations to the Haar functions are stable (in the Bessel bound sense) with respect to small errors in dilation and translation and can be replaced by fine discretizations without producing too much additional error.