Spatial Dynamics of Evolution Systems in Ecology and Epidemiology
Dynamiques spatiales des systèmes évolutifs en écologie et en épidémiologie
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ELENA BRAVERMAN, University of Calgary
On the interplay of harvesting and various diffusion strategies for spatially heterogeneous populations

We study a Lotka system describing two competing populations, and each of them chooses its diffusion strategy as the tendency to have a distribution proportional to a certain positive prescribed function. For instance, the standard diffusion corresponds to the choice of a uniform distribution. We focused on the interplay of species competition and diffusion strategies with some other factors incorporated: different growth rates, carrying capacities and harvesting. We describe the cases when the choice of diffusion strategies promotes coexistence.

In the case of populations exploitation, the harvesting effort is assumed to be proportional to the space-dependent intrinsic growth rate. The differences between the two populations are the diffusion strategy and the harvesting intensity. In the absence of harvesting, competing populations may either coexist, or one of them may bring the other to extinction. If the latter is the case, introduction of any level of harvesting to the successful species guarantees survival to its non-harvested competitor. In the former case, there is a strip of "close enough" to each other harvesting rates leading to preservation of the original coexistence. Some estimates are obtained for the relation of the harvesting levels providing either coexistence or competitive exclusion.

YUMING CHEN, Wilfrid Laurier University
Global dynamics of a diffusive SEICR HCV model with nonlinear incidences

Considering acute and chronic infections, we propose and study the spatiotemporal dynamics of a reaction-diffusive SEICR model for hepatitis C infection. The well-posedness and boundedness of solutions are established. The model possesses a threshold dynamics characterized by the basic reproduction number $\mathcal{R}_0$. When $\mathcal{R}_0 < 1$ the disease free equilibrium is globally asymptotically stable while the system is uniformly persistent when $\mathcal{R}_0 > 1$. In the special case of homogeneous space with heterogeneous diffusion, the explicit expression of $\mathcal{R}_0$ is derived. Moreover, if $\mathcal{R}_0 > 1$, the system has a unique homogeneous endemic equilibrium, which is globally asymptotically stable. The theoretical results are illustrated with numerical simulations. Sensitivity of $\mathcal{R}_0$ on the parameters is also carried out.

JIAN FANG, Harbin Institute of Technology
Optimal dispersal strategy for a stage-structured alien species in periodic discrete habitat

Starting from an age-structured diffusive population growth law in a discrete and periodic habitat, we formulate a stage-structured population model for a single alien species with spatially periodic dispersal, mortality and recruitment. Then we discuss the optimal dispersal strategy to maximize the invasion speed in different biological scenarios.

TAHIR BACHAR ISSA, San Jose State University
Pointwise persistence in full chemotaxis models with logistic source on bounded heterogeneous environments

In this talk, I will present our work on pointwise persistence in full chemotaxis models with local as well as nonlocal time and space dependent logistic source in bounded domains. We first prove the global existence and boundedness of nonnegative classical solutions under some conditions on the coefficients in the models. Next, under the same conditions on the coefficients, we show that pointwise persistence occurs, that is, any globally defined positive solution is bounded below by a positive constant independent of its initial condition when the time is large enough. It should be pointed out that Tao and Winkler in 2015, established the persistence of mass for globally defined positive solutions, which indicates that any extinction phenomenon, if occurring at all, necessarily must be spatially local in nature, whereas the population as a whole always persists. The pointwise persistence proved in this work implies that not only the population as a whole persists, but also it persists at any location eventually. It also implies the existence of strictly positive entire solutions.

ADRIAN LAM, The Ohio State University
Asymptotic spreading of KPP reactive fronts in heterogeneous shifting environments

We study the asymptotic spreading of Kolmogorov-Petrovsky-Piskunov (KPP) fronts in heterogeneous shifting habitats, with any number of shifting speeds, by further developing the method based on the theory of viscosity solutions of Hamilton-Jacobi equations. Our framework addresses both reaction-diffusion equation and integro-differential equations with a distributed time-delay. The latter leads to a class of limiting equations of Hamilton-Jacobi-type depending on the variable $s = x/t$ and in which the time and space derivatives are coupled together. We will first establish uniqueness results for these Hamilton-Jacobi equations using elementary arguments, and then characterize the spreading speed in terms of a reduced equation on a one-dimensional domain in the variable $s = x/t$. In terms of the standard Fisher-KPP equation, our results lead to a new class of "asymptotically homogeneous" environments which share the same spreading speed with the corresponding homogeneous environments. This is joint work with Xiao Yu of South China Normal University.

KUNQUAN LAN, Ryerson University
Fixed point index theory for r-nowhere normal-outward compact maps and logistic type population models

This presentation is based on recent work on the fixed point index theory for r-nowhere normal-outward compact maps obtained by Yang and Lan in 2016, Hammerstein integral equations, boundary value problems and applications to logistic type population models.

MARK LEWIS, University of Alberta
Mathematical models for the neutral genetics of changing populations

In this talk I will discuss the genetic structure of populations subject to climate change and undergoing range expansion. The models and analyses are based on reaction diffusion and integrodifference equations for the asymptotic neutral genetic structure of populations. We decompose solutions into neutral genetic components called neutral fractions. The "inside dynamics" then describe the spatiotemporal evolution of these fractions and can be used to predict changes in genetic diversity. Extensions are made to include stage-structure in the population dynamics and mutations in the genetic fractions. Results are compared with small-scale experimental systems that have been developed to test the mathematical theory. This work is joint with Nathan Marculis, Roger Lui and Jimmy Garnier

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FRITHJOF LUTSCHER, University of Ottawa
Population dynamics in fragmented landscapes: models, results, and future challenges

Reaction-diffusion equations have been the workhorse for modelling and analyzing population dynamics in space and time for many decades. The most famous result is the calculation of an invasion speed that goes back to Fisher and Kolmogorov, Petrovsky and Piskunov in the 1930s. As ecologists increasingly acknowledged the importance of different habitat types and landscape fragmentation for species persistence and spread, modellers began to include such elements into reaction-diffusion equations, and to analyze the resulting properties. These efforts have led to a wealth of models and mathematical results since the hallmark paper by Shigesada, Kawasaki and Teramoto in 1986. In this talk, I will review some of the ecological questions, the mathematical models, and their results from the past two decades. I will point out similarities and differences and some challenges that future models and analysis should address.

MING MEI, Champlain College St-Lambert
Sharper traveling waves for degenerate Burgers equations with time-delay

In this talk, I will present some recent results of study on Burgers equations with time-delay and degeneracy of viscosity. The modelling equations arise from the nonlocal reaction-diffusion equations for population dynamics. When the Rankine-Hugoniot condition and the Lax's entropy condition hold, the dynamical equation possesses some sharper traveling waves, the so-called sharper viscous shock waves. The degeneracy of viscosity causes the traveling waves to be sharper, and the large time-delay makes the traveling waves to be oscillatory.

CHUNHUA OU, Memorial University of Newfoundland
Determinacy of the Single Spreading Speed or Multiple Spreading Speeds for a Cooperative Lotka-Volterra System

Cooperation in population systems can result in the existence of a co-existence (win-win) equilibrium. When diffusion is incorporated, individual species possibly invade into the far end with different spreading speeds. Predicting or determining them (including the fast and slow spreading speeds) becomes challenging. This talk is devoted to the determinacy of invasion speed of each species for a cooperative Lotka-Volterra system, which admits single or multiple spreading speeds (co-speed or fast-slow speed). In the existence of a single spreading speed, the two species share a common invasion speed, and nonnegative traveling wave profiles, connecting the extinction equilibrium, exist, if the wave speed is not less than the common speed. Predicting or determining the invasion speed can be linked to the linearized system at the extinction state. The existence of multiple spreading speeds indicates new connections of traveling wave profiles into certain intermediate states. Due to this, the determinacy of each spreading speed focuses not only on the extinction states but also on the corresponding intermediate states. Based on constructions of upper-lower solutions, we establish criteria that can determine the fast-slow invasive speeds. Our speed selection mechanism can also help scientists greatly understand the movement of stacked fronts in cooperative systems with multiple species.

SHIGUI RUAN, University of Miami
Modeling the Growth, Invasion and Competition of Aedes Mosquitoes in Florida

The Aedes mosquitoes, in particular Aedes aegypti and Aedes albopictus, are the primary vectors that transmit several arboviral diseases, including chikungunya fever, dengue fever, yellow fever, and Zika. Recently, the world has been experiencing a series of major outbreaks of these vector-borne diseases (for example, the 2016 Zika outbreak in Florida, etc.). In order to study the transmission dynamics of these vector-borne diseases, it is very important and necessary to understand the population dynamics, current distributions and movements of Aedes mosquitoes for successful surveillance and control programs. In this talk, we will introduce some of our recent studies on modeling the population dynamics of Aedes mosquitoes, the invasion of Aedes albopictus mosquitoes, and the competition between Aedes aegypti and Aedes Albopictus mosquitoes in Florida, the United States. In particular, we propose a competition model with road-field diffusion in which the invasive population not only disperses in the interior of the spatial domain but also moves faster on the boundary of the domain. Both strong-weak and weak-weak competitions are discussed. It is shown that the asymptotic spreading speed of the wave fronts is increasing only if the road diffusion rate is greater than the field diffusion rate. Numerical simulations are presented to illustrate our analytical results and to explain the current estimated distributions of these two mosquito species in Florida.

RACHIDI SALAKO, University of Nevada, Las Vegas
On the effect of lowering population's movement to control the spread of an infectious disease

We study the asymptotic behavior of endemic equilibrium solutions of a diffusive infection epidemic model in spatial heterogeneous environment when the diffusion rate d_S of the susceptible hosts and the diffusion rate d_I of the infected group of the population are sufficiently small. In particular, we address the question of how the magnitude of the ratio $\frac{d_I}{d_S}$ affects the total size of the infected group at endemic equilibrium when both d_S and d_I are sufficiently small. Our results indicate that when d_I and d_S are sufficiently small, the size of $\frac{d_I}{d_S}$ plays a crucial role in the dynamics of the disease in the sense that : (i) if $\frac{d_I}{d_S}$ is sufficiently small, the disease may persist and the total size of the infected group will approach its maximal size; (ii) if $\frac{d_I}{d_S}$ is significantly large, then the total size of the susceptible hosts is maximized while the total size of the infected group is minimized. Hence, our results suggest that lowering the movement rate of the population in the attempt to control the spread of an infectious disease is an effective control strategy if the susceptible hosts movement's rate is kept sufficiently smaller than that of the infected individuals.

WENXIAN SHEN, Auburn University
Population dynamics under climate change: persistence criterion and effects of fluctuations

The current talk is concerned with population dynamics under climate change. The evolution of species is modelled by a reaction-diffusion equation in a spatio-temporally heterogeneous environment described by a climate envelope that shifts with a time-dependent speed function. For a general almost-periodic speed function, we establish the persistence criterion in terms of the sign of the approximate top Lyapunov exponent and, in the case of persistence, prove the existence of a unique forced wave solution that dominates the population profile of species in the long run. In the setting for studying the effects of fluctuations in the shifting speed or location of the climate envelope, we show by means of matched asymptotic expansions and numerical simulations that the approximate top Lyapunov exponent is a decreasing function with respect to the amplitude of fluctuations, yielding that fluctuations in the shifting speed or location have negative impacts on the persistence of species, and moreover, the larger the fluctuation is, the more adverse the effect is on the species. In addition, we assert that large fluctuations can always drive a species to extinction. Our numerical results also show that a persistent species under climate change is invulnerable to mild fluctuations, and becomes vulnerable when fluctuations are so large that the species is endangered.

ZHONGWEI SHEN, University of Alberta
Spreading speeds in random environments

Integrodifference equations have been widely used to model the invasion of species, the spread of diseases, etc. Traditional models assume the environment is temporally constant. A class of integrodifference equations with random coefficients is studied to understand the consequences of random fluctuations. In this talk, I will present some results about the strategy for establishing the spreading speed and the effects of random fluctuations on the spreading speed.

JUNPING SHI, College of William & Mary
Spatial movement with diffusion and memory-based self-diffusion and cross-diffusion

Spatial memory has been considered significant in animal movement modeling. We formulate a two-species interaction model by incorporating both random walk and spatial memory-based walk in their movement. The spatial memory-based walk, described by a chemotactic-like term, is derived by a modified Fick's law involving a directed movement toward the gradient of the density distribution function at a past time. For the proposed model, local stability and bifurcations are studied at constant steady states. Unlike a classical reaction-diffusion equation, we show that the accumulation points of eigenvalues for the model will locate at a vertical line in the complex plane, which will make the model generate spatially inhomogeneous time-periodic patterns through Hopf bifurcation. As illustrations, we apply these results to competition and cooperative models with memory-based diffusion. For the competition model, it turns out that the outcomes are far more complicated than those of classic Lotka-Volterra reaction-diffusion models, due to the consideration of memory-based diffusion. In particular, the existence of periodic oscillations is proved under weak competition. Similar conclusions hold for the cooperative model. This is a joint work with Chuncheng Wang and Hao Wang.

MARCO TOSATO, York University
A Patchy Model for Tick-borne Disease Spread with Patch-Specific Developmental Delays

We introduce a two-patch model with multiple delays to describe how tick population dynamics is affected by host mobility and local environmental factors.

In this talk, I will start by giving a brief introduction on ticks with particular interest on their lifecycle, suitable environments for their development and possible tick-control strategies. Then, I will describe the model and explain how the dynamical behaviors depend on patch-specific basic reproduction numbers and host mobility by using singular perturbation analyses and monotone dynamical systems theory. Finally, I will discuss how these results might provide useful insights for tick population control strategies.

This is a joint work with Prof. Xue Zhang and Prof. Jianhong Wu.

HAO WANG, University of Alberta
A new free boundary problem

I will present a novel free boundary problem to model the movement of single species with a range boundary. The change of a free boundary is assumed to be influenced by the weighted total population inside the range boundary, which is described by an integro-differential equation. Our free boundary equation is a generalization of the classical Stefan condition that allows for nonlocal influences on the boundary movement. We prove that the new model is well posed and possesses steady state. The spreading speed of the model is smaller than that for the equivalent problem with a Stefan condition. While the classical Stefan condition categorizes asymptotic behavior via a spreading-vanishing dichotomy, the new model extends this dichotomy to a spreading-balancing-vanishing trichotomy. Our model allows both expansion and shrinking of the range boundary. When the model is extended to have two free boundaries, we observe asymmetric shifts, as well as steady state within synchronous moving boundaries. These are newly discovered phenomena in free boundary problems of animal movement.

*This is a joint work with Mark Lewis, Chuncheng Wang, and Chunxi Feng.

LIN WANG, University of New Brunswick
Effects of intraguild prey dispersal driven by intraguild predator-avoidance on species coexistence

In this talk, I will present a novel mathematical model that couples a competition model with an intraguild predation model via dispersal of intraguild prey driven by intraguild predator-avoidance. We show that a large dispersal rate would lead to the collapse of species coexistence, which is consistent with the reported experimental results. In addition, we show that three modes of species coexistence are possible when the intraguild prey dispersal rate is not too large. Moreover, for a certain range of dispersal rates, a stable interior equilibrium can coexist with a stable positive limit cycle.

XIANGSHENG WANG, University of Louisiana at Lafayette
Global analysis of a viral infection model with cell-to-cell transmission and immune chemokines

In this talk, we study a viral infection model incorporating both cell-to-cell infection and immune chemokines. Based on experimental results in the literature, we make a basic assumption that the cytotoxic T lymphocytes (CTL) will move toward the location with more infected cells, while the diffusion rate of CTL is a decreasing function of the density of infected cells. We first establish the global existence and ultimate boundedness of the solution via a priori energy estimates. Next, we define the basic reproduction number of viral infection R_0 and prove by uniform persistence theory, Lyapunov functional technique, and LaSalle invariance principle that the infection-free steady state E_0 is globally asymptotically stable if $R_0 < 1$. When $R_0 > 1$, then E_0 becomes unstable, and another basic reproduction number of CTL response R_1 becomes the dynamic threshold in the sense that, if $R_1 < 1$, then the CTL-inactivated steady state E_1 is globally asymptotically stable; and if $R_1 > 1$, then the CTL-activated steady state E_2 is globally asymptotically stable.

XUEYING WANG, Washington State University
Target reproduction numbers for reaction-diffusion population models

A very important population threshold quantity is the target reproduction number, which is a measure of control effort required for a target prevention, intervention or control. This concept, as a generalization of type reproduction number, was first introduced in Shuai et al. (J Math Biol 67:1067–1082, 2013) for nonnegative matrices with immediate applications to compartmental population models of ordinary differential equations. The current paper is devoted to the study of all target reproduction numbers for reaction-diffusion population models with compartmental structure. It turns out that the target reproduction number can be regarded as the basic reproduction number of a modified system, where the state of newborn individuals is limited to the target control set and the offspring from the non-target set is regarded as a part of the transition. In other words, the target reproduction number can be interpreted as the expected number of offspring in a specific target set that a primary newborn individual of the same set would produce during its lifetime. We also characterize the target reproduction number so that it can be easily computed numerically for reaction-diffusion models. At the end, we demonstrate our theoretical observations using two examples.

XIAOXIA XIE, Idaho State University

Modeling the transmission dynamics of dengue in the presence of Wolbachia with delay differential euquations

Dengue is a serious concern in many parts of the world and dengue prevention relies primarily on vector control but the failure of traditional methods has promoted the development of novel entomological approaches, such as the intracellular bacterium Wolbachia to control mosquito populations, which has gained significant interest as a potential agent of dengue control in the last decade. Here, a system of delayed differential equations is developed to illustrate the efficiency of Wolbachia intervention.

SHUWEN XUE, Memorial University of Newfoundland

Persistence, spreading speeds and forced waves of Parabolic–Elliptic Chemotaxis models in shifting environments

Chemotaxis models are used to describe the movements of biological species or living organisms in response to certain chemicals in their environments. The current talk is concerned with persistence, spreading speeds and forced waves of Parabolic–Elliptic Chemotaxis models in shifting environments. Some numerical simulations will be presented to demonstrate the existence of forced waves.

CHAYU YANG, University of Nebraska - Lincoln

Basic reproduction numbers for a class of reaction-diffusion epidemic models

We study the basic reproduction numbers for a class of reaction-diffusion epidemic models that are developed from autonomous ODE systems and present a general numerical framework to compute such basic reproduction numbers; meanwhile, the numerical formulation provides useful insight into their characterizations. Using matrix analysis, we show that the basic reproduction numbers are the same for these PDE models and their associated ODE models in several important cases that include, among others, a single infected compartment, constant diffusion rates, uniform diffusion patterns among the infected compartments, and partial diffusion in the system.

AIJUN ZHANG, University of Louisiana

A Discrete-time Predator-Prey Model with Selection and Mutation

We study a discrete-time predator-prey system with selection and mutation in the prey population where individuals are distributed over a finite number of phenotypic traits. For the pure selection case, we establish conditions for competitive exclusion between individuals with different traits in the prey population and we show that the system converges to a boundary equilibrium representing the predator and the fittest prey trait. For the full selection mutation model, we explore the coexistence and persistence. Finally, we offer some examples of numerical simulation.

LEI ZHANG, Harbin Institute of Technology at Weihai

Propagation dynamics of reaction-diffusion equations in a time-heterogeneous shifting environment

In this paper, we study the propagation dynamics of a large class of time and space heterogeneous reaction-diffusion equations

$$u_t = u_{xx} + ug(x - \omega(t), t, u),\ x \in \mathbb{R},$$

where $\omega(t)$ represents the shifting distance, and the nonlinearity $ug(\xi, t, u)$ is asymptotically of KPP type as $\xi \to -\infty$ and is negative as $\xi \to +\infty$. Let c^* be the spreading speed of the limiting equation $u_t = u_{xx} + ug(-\infty, t, u)$. Under the assumption that the shifting speed $\omega'(t)$ admits a uniform mean c, we show that the solutions with compactly supported initial data go to zero eventually when $c \leq -c^*$, the leftward spreading speed is $-c^*$ when $c > -c^*$, and the rightward spreading speed is c and c^* when $c \in (-c^*, c^*)$ and $c \geq c^*$, respectively. We also establish the existence, uniqueness and nonexistence of the forced traveling wave in terms of the sign of $c - c^*$. This talk is based on a joint work with Prof. Xiao-Qiang Zhao.

XINGFU ZOU, University of Western Ontario
Evolution and adaptation of anti-predation response of prey in a two-patchy environment

When perceiving a risk from predators, a prey may respond by reducing its reproduction and decreasing or increasing (depending on the species) its mobility. We formulate a patch model to investigate the aforementioned fear effect which is indirect, in contrast to the predation as a direct effect, on the prey population. We consider not only cost but also benefit of anti-predation response of the prey, and explore their trade-offs as well as the impact of the fear effect mediated dispersals of the prey. In the case of constant response level, if there is no dispersal and for some given response functions, the model indicates the existence of an evolutionary stable strategy (ESS) which is also a convergence stable strategy (CSS) for the response level; and if there is dispersal, the analysis of the model shows that it will enhance the co-persistence of the prey on both patches. Considering the trait as another variable, we continue to study the evolution of anti-predation strategy for the model with dispersal, which leads to a three-dimensional system of ordinary differential equations. We perform some numerical simulations, which demonstrate global convergence to a positive equilibrium with the response level evolving toward a positive constant level, implying the existence of an optimal anti-predation response level. Interestingly, it is observed that this optimal response level may not agree with the ESS. This is a joint work with Ao Li.