Recent advancement in Nonlinear Partial Differential Equations
Avancées récentes dans les équations aux dérivées partielles non-linéaires

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ANDREW COLINET, McMaster University

Solutions of the Ginzburg-Landau equations with vorticity concentrating near a nondegenerate geodesic

It is well-known that under suitable hypotheses, for a sequence of solutions of the (simplified) Ginzburg-Landau equations $-\Delta u_\varepsilon + \varepsilon^{-2}(|u_\varepsilon|^2 - 1)u_\varepsilon = 0$, the energy and vorticity concentrate as $\varepsilon \to 0^+$ around a codimension 2 stationary varifold — a (measure theoretic) minimal surface. Much less is known about the question of whether, given a codimension 2 minimal surface, there exists a sequence of solutions for which the given minimal surface is the limiting concentration set. The corresponding question is very well-understood for minimal hypersurfaces and the scalar Allen-Cahn equation, and for the Ginzburg-Landau equations when the minimal surface is locally area-minimizing, but otherwise quite open.

We consider this question on a 3-dimensional closed Riemannian manifold (M, g), and we prove that any embedded nondegenerate closed geodesic can be realized as the asymptotic energy/vorticity concentration set of a sequence of solutions of the Ginzburg-Landau equations.

XINYANG LU, Lakehead University

Estimates on a convex singular potential for nematic liquid crystals

Liquid crystals (LC) are an intermediate state of the matter between solids and liquids, They exhibit significant mobility, but differently from pure liquids, they have a preferred orientation, commonly referred to as "director". They appear from a wide variety of fields, such as electronics, biology, virology, etc. LCs themselves exhibit several phases, e.g. nematic, smectic, chiral/twisted, discotic, conic. Nematic LCs are the simplest ones, being characterized by only a director, without any polarization or more complicated structures.

Modeling LCs has been a long standing problem. One of the most widely models is the Landau-de Gennes theory. The main quantity is a 3×3 Q-tensor matrix, and the associated energy is composed of an elastic and a bulk part. Due to modeling requirements, the eigenvalues of the Q-tensor must be constrained in $(-1/3, 2/3)$, a condition known as "physicality". One way to enforce this is to add a convex singular potential ψ. Powerful from a theoretical point of view, such ψ is defined only implicitly, making its analysis quite challenging. In this talk, we will present several crucial estimates on ψ, and its derivatives.

PRITPAL MATHARU, McMaster University

PDE Optimization for Theoretical and Computational Turbulence Problems

Turbulent flows occur in various fields and are a central, yet extremely complex topic in fluid dynamics. Understanding the behaviour of fluids is vital for multiple research areas including, but not limited to: biological models, weather prediction, and engineering design models for automobiles and aircrafts. In this talk, we will introduce PDE optimization techniques utilizing adjoint calculus and computational framework to be used in suitable gradient-based techniques. Then, we will discuss how these techniques and their modifications have been employed for problems in both computational and theoretical turbulence problems, concerning the turbulence closure problem and the "zeroth law of turbulence".

LORENA AGUIRRE SALAZAR, VSU

On TFDW-type models and an Ohta-Kawasaki model

The Thomas-Fermi-Dirac-Von Weizsäcker (TFDW) model is a physical model describing ground-state electron configurations of many-body systems. On the other hand, the Ohta-Kawasaki model arises in the context of diblock copolymer melts. In this talk, we discuss results concerning the compactness of minimizing sequences of TFDW-type models and a variant of the Ohta-Kawasaki model.

YURIJ SALMANIW, University of Alberta

Habitat Loss: Bridging the gap between habitat degradation and habitat destruction in a competitive reaction-diffusion system

In this talk I will introduce a diffusive competition model with habitat degradation and homogeneous Neumann boundary conditions in a bounded domain that is partitioned into the healthy region (undisturbed habitat) and the degraded region (due to anthropogenic habitat disturbance). Species follow the Lotka-Volterra competition in the healthy region while in the degraded region species experience only exponential decay (not necessarily at the same rate). Using theory from monotone dynamical systems and other related results, a complete description of the global dynamics will be presented. I will then introduce a habitat destruction problem and make rigorous the connection between these two formulations. It is found that the destruction problem can be viewed as a limiting case of the degradation problem. This requires some interesting convergence results between two eigenvalue problem formulations and the related scalar equation, which in turn provides significant insight into the destruction competition system. I will conclude with some future directions currently being investigated.

SZYMON SOBIESZEK, McMaster University

Ground states in the energy-supercritical Gross-Pitaevskii equation with a harmonic potential

The energy super-critical Gross–Pitaevskii equation with a harmonic potential is revisited in the case of cubic focusing nonlinearity and dimension $d \geq 5$. In order to prove the existence of a ground state (a positive, radially symmetric solution in the energy space), we develop the shooting method and deal with a one-parameter family of classical solutions to an initial-value problem for the stationary equation. We prove that the solution curve (the graph of the eigenvalue parameter versus the supremum norm) is oscillatory for $d \leq 12$ and monotone for $d \geq 13$. Compared to the existing literature, rigorous asymptotics are derived by constructing different families of solutions to the stationary equation with functional-analytic rather than geometric methods.

IHSAN TOPALOGLU, Virginia Commonwealth University

Riesz-type inequalities and overdetermined problems for triangles and quadrilaterals

In this talk I will consider Riesz-type nonlocal interaction energies over convex polygons. After proving the analog of the Riesz inequality in this discrete setting for triangles and quadrilaterals, and obtaining that among all N-gons with fixed area, the nonlocal energy is maximized by a regular polygon, for $N = 3, 4$, I will present on the necessary first-order stationarity conditions for a polygon with respect to a restricted class of variations, which is then used to characterize regular N-gons, for $N = 3, 4$, as solutions to an overdetermined free boundary problem. This is a joint project with Marco Bonacini and Riccardo Cristoferi.

LEE VAN BRUSSEL, McMaster University

An orthogonality condition for minimizers of a Ginzburg-Landau functional

In this work, minimizers of the Ginzburg-Landau functional with associated length scale parameter $\varepsilon > 0$ are studied over a bounded simply-connected domain $\Omega \subset \mathbb{R}^2$ with smooth boundary $\partial\Omega$. Along the boundary, minimizers u_ε are to satisfy the orthogonality condition $\langle u_\varepsilon, g^\perp \rangle = 0$ where g is a smooth $\mathbb{S}^1$-valued function of degree $d \in \mathbb{Z} \setminus \{0\}$ defined on $\partial\Omega$. We will discuss some properties of minimizers and analyze their limiting behaviour along a subsequence $\varepsilon_n \to 0$.

RAGHAV VENKATRAMAN, Courant Institute

Least Wasserstein Distance Between Disjoint Shapes With Perimeter Regularization

We prove the existence of global minimizers to the double minimization problem

$$\inf\left\{P(E)+\lambda W_p(\mathcal{L}^n\lfloor E,\mathcal{L}^n\lfloor F)\colon |E\cap F|=0,\,|E|=|F|=1\right\},$$

where $P(E)$ denotes the perimeter of the set E, W_p is the p-Wasserstein distance between Borel probability measures, and $\lambda>0$ is arbitrary. The result holds in all space dimensions, for all $p\in[1,\infty)$, and for all positive λ. This answers a question of Buttazzo, Carlier, and Laborde.

CHONG WANG, Washington and Lee University

Modeling and Analysis of Patterns in Multi-Constituent Systems with Long Range Interaction

Skin pigmentation, animal coats and block copolymers can be considered as multi-constituent inhibitory systems. Exquisitely structured patterns arise as orderly outcomes of the self-organization principle. The free energy functional combines an interface energy favoring micro-domain growth with a Coulomb-type long range interaction energy which prevents micro-domains from unlimited spreading. Analytically, via the sharp interface model, we study the exact shape of global minimizers and also construct stable stationary sets. Numerically, via the diffuse interface model, one open question related to the polarity direction of double bubble assemblies is answered. Moreover, many quantitative results are proved both numerically and theoretically.