Recent Developments in Complex Geometry and Geometric Analysis
Développements récents en géométrie complexe et en analyse géométrique
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MICHAEL ALBANESE, UQAM

The Yamabe Invariant of Complex Surfaces

The Yamabe invariant is a real-valued diffeomorphism invariant coming from Riemannian geometry. Using Seiberg-Witten theory, LeBrun showed that the sign of the Yamabe invariant of a Kähler surface is determined by its Kodaira dimension. We consider the extent to which this remains true when the Kähler hypothesis is removed. This is partly based on joint work with Claude LeBrun.

JINGYI CHEN, University of British Columbia

Stability of the fibres of Hopf surfaces as harmonic maps and minimal surfaces

We construct a family of Hermitian metrics on the Hopf surface $S^3 \times S^1$, whose fundamental classes represent distinct cohomology classes in the Aeppli cohomology group. These metrics are locally conformally Kahler. Among the toric fibres of $\pi : S^3 \times S^1 \to CP^1$ two of them are stable minimal surfaces and each of the two has a neighbourhood so that fibres therein are given by stable harmonic maps from 2-torus and outside, far away from the two tori, there are unstable harmonic ones that are also unstable minimal surfaces. A similar result is true for $S^{2n-1} \times S^1$.

DAREN CHENG, University of Waterloo

Existence of constant mean curvature 2-spheres

Constant mean curvature (CMC) surfaces arise in many different contexts and are natural generalizations of minimal surfaces. An important question is finding CMC surfaces with controlled topology in 3-manifolds. In this talk, I'll describe joint work with Xin Zhou (Cornell), in which we address the genus-zero case, where the surface sought after is a sphere. Our main result is that in an arbitrary Riemannian 3-sphere, for almost every H there exists a branched immersed 2-sphere with constant mean curvature H. Moreover, the existence extends to all H when the Riemannian 3-sphere has positive Ricci curvature.

TRISTAN COLLINS, MIT

SYZ mirror symmetry for some Calabi-Yau surface pairs

I will discuss a proof of a strong form of the SYZ mirror symmetry conjecture for Calabi-Yau surfaces pairs constructed from del Pezzo surfaces and rational elliptic surfaces. Time permitting, I will also mention some applications, including to the Torelli theorem for ALH* gravitational instantons. This is joint work with A. Jacob and Y.-S. Lin.

RONAN CONLON, The University of Texas at Dallas

Kähler-Ricci solitons

A Kähler-Ricci soliton is a solution of the Kähler-Ricci flow that evolves only by scaling and the action of biholomorphisms generated by the flow of a real holomorphic vector field. They may model the formation of singularities of the flow. I will give an overview of recent work with Deruelle and Deruelle-Sun on these objects.

BIN GUO, Rutgers University - Newark

Stability estimates for the complex Monge-Ampere and Hessian equations

We will present a new proof for stability estimates for the complex Monge-Ampère and Hessian equations, which does not require pluripotential theory. This is based on joint work with D.H. Phong and F. Tong.

SPIRO KARIGIANNIS, University of Waterloo
Variational characterization of certain calibrated submanifolds

Fix a compact, oriented, embedded submanifold L of a manifold M. Consider the volume $\mathcal{V}(g)$ of L as a functional of the ambient Riemannian metric g on M. We show that when g is induced from a special geometric structure (specifically a $\mathrm{U}(m)$, a G_2, or a $\mathrm{Spin}(7)$ structure) and is varied only in a particular special way, then g is a critical point of $\mathcal{V}$ if and only if L is a calibrated submanifold of M. This generalizes a result of Arezzo-Sun (which was established only for Kahler manifolds) to a much wider class of special ambient geometries, with no assumption on torsion. The $\mathrm{Spin}(7)$ case is particularly interesting, as it behaves somewhat differently from the other cases. This is joint work in progress with Da Rong Cheng and Jesse Madnick.

JULIEN KELLER, UQAM
Constant scalar curvature Kähler cone metrics

We will review some recent progress on constant scalar curvature Kähler metrics with conical singularities along a divisor. We will discuss the existence of such special metrics, focussing on the particular case of projective bundles. Moreover, we will explain some very recent results on the relationship between such metrics and uniform log-K stability in view of the logarithmic Yau-Tian-Donaldson conjecture.

MAN CHUN LEE, The Chinese University of Hong Kong
continuous metrics and scalar curvature

A classical Theorem in conformal geometry states that on a closed manifold with non-positive Yamabe invariant, a smooth metric achieving the invariant must be Einstein. It is conjectured by Schoen that the same conclusion still hold if a metric is uniform bi-Lipschitz and is smooth away from a co-dimension three sub-manifold. In this talk, we will discuss some of the recent progress toward the conjecture. This is a joint work with L.-F. Tam.

CHAO MING LIN, University of California-Irvine
The deformed Hermitian—Yang—Mills equation, the family of C-subsolutions, and the solvability

The deformed Hermitian—Yang—Mills equation, which will be abbreviated as dHYM equation, was discovered around the same time in the year 2000 by Mariño—Minasian—Moore—Strominger and Leung—Yau—Zaslow using different points of view.

In this talk, first, I will skim through Leung—Yau—Zaslow's approach in a simple way. Then I will introduce the C-subsolution which is introduced by Székelyhidi (See also Guan) and I will go over some known solvability results of the dHYM equation. Last, I will show some of my recent works on the solvability when complex dimension equals three or four.

ALEXANDRA OTIMAN, University of Florence
New constructions in non-Kähler toric geometry

In this talk I plan to describe a special class of Kato manifolds in arbitrary complex dimension, whose construction arises from toric geometry. Using the toric language, I will present several of their analytic and geometric properties, including existence of special complex submanifolds and partial results on their Dolbeault cohomology. Moreover, since they are compact complex manifolds of non-Kahler type, I will investigate what special Hermitian metrics they support.

SISI SHEN, Columbia University
A Chern-Calabi flow on Hermitian Manifolds

We discuss the existence problem of constant Chern scalar curvature metrics on a compact complex manifold and introduce a Hermitian analogue of the Calabi flow on compact complex manifolds with vanishing first Bott-Chern class.

FREID TONG, Harvard University
Uniform estimates for complex Monge-Ampere equations

The complex Monge-Ampere equation occupies a central role in Kähler geometry, and the most subtle part of the theory of complex Monge-Ampere equations is perhaps the uniform estimate. In this talk, I will introduce the complex Monge-Ampere equation and discuss a new approach to establish sharp uniform estimates, this new method is very versatile and has the advantage of extending to a large family of fully non-linear equations. This is based on joint work with B. Guo and D.H. Phong.

YURY USTINOVSKY, Lehigh University
Variational approach to the steady Generalized Kähler-Ricci solitons

This talk concerns Generalized Kahler-Ricci solitons (GKRS) - geometric structures independently arising in the context of supersymmetric sigma models, generalized geometry of Hitchin and Gualtieri, uniformization problems in non-Kahler geometry and Einstein-Weyl geometry. GKRS are natural generalizations of Ricci-solitons, incorporating a torsion term.

Every Generalized Kahler (GK) structure on a smooth manifold M admits an infinitesimal variation parametrized by $C^\infty(M, R)/R$. Integrating such infinitesimal variations we obtain a notion of GK class, and it makes sense to study existence and uniqueness questions for GKRS in a given class. Following the fundamental ideas in Kahler geometry, we define a weighted J-functional on the GK class of a log-nondegenerate GK manifold. This functional naturally extends the Aubin' functional in Kahler geometry, and J-functional of Apostolov and Streets in nondegenerate GK setting. We prove that a log-nondegenerate GK structure is a critical point for the weighted J-functional if and only if it is a gradient steady GKRS. We use this functional to prove rigidity of solitons in a given GK class, and use the latter to deduce complete classification of compact GKRS for $\dim M = 4$ (joint with V.Apostolov and J.Streets)