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Perfect and almost perfect linear Lee codes

Given a positive integer r, an abelian group G and a subset $T = \{a_1, a_2, \cdots, a_n\} \subseteq G \setminus \{e\}$, if all elements in the multiset

$$\Psi := \left\{ * \; a_1^{\pm j_1} \cdots a_n^{\pm j_n} : 0 \leq \sum_{k=1}^{n} j_k \leq r, j_k \in \mathbb{Z}_{\geq 0} \; * \right\}$$

are distinct, and $G = \Psi$, then we call the Cayley graph $\Gamma(G, S)$ an *Abelian-Cayley-Moore graph*, where $S := T \cup T^{(-1)}$. Under this condition, the size of G (i.e. $|\Psi|$) is $\sum_{i=0}^{\min\{n,r\}} 2^i \binom{n}{i}\binom{r}{i}$.

It is a bit surprising that the existence of an Abelian-Cayley Moore graph is equivalent to a perfect linear Lee code of radius r in $\mathbb{Z}^n$, that is a lattice tiling of $\mathbb{Z}^n$ by the translations of an ℓ_1-metric sphere of radius r. More than 50 years ago, Golomb and Welch conjectured that there is no perfect Lee code C for $r \geq 2$ and $n \geq 3$. Recently, Leung and the speaker proved that if C is linear, then Golomb-Welch conjecture is true for $r = 2$ and $n \geq 3$.

In this talk, we consider the classification of linear Lee codes of the second best possibility, that is the density of the lattice packing of $\mathbb{Z}^n$ by Lee spheres $S(n, r)$ equals $\frac{|S(n,r)|}{|S(n,r)|+1}$. By checking the corresponding abelian Cayley graphs, an almost perfect linear Lee code is equivalent to the case with $G = \Psi \cup \{f\}$ where f is the unique element of order 2 in G.