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Bias in cubic Gauss sums

We prove, in this joint work with Maksym Radziwill, a 1978 conjecture of S. Patterson (conditional on the Generalised Riemann hypothesis) concerning the bias of cubic Gauss sums. This explains a well-known numerical bias in the distribution of cubic Gauss sums first observed by Kummer in 1846.

There are two important byproducts of our proof. The first is an explicit level aspect Voronoi summation formula for cubic Gauss sums, extending computations of Patterson and Yoshimoto. Secondly, we show that Heath-Brown's cubic large sieve is sharp under GRH. This disproves the popular belief that the cubic large sieve can be improved.

An important ingredient in our proof is a dispersion estimate for cubic Gauss sums. It can be interpreted as a cubic large sieve with correction by a non-trivial asymptotic main term. This estimate relies on the Generalised Riemann Hypothesis, and is one of the fundamental reasons why our result is conditional.