ALEX BARRIOS, Carleton College
Elliptic curves with non-trivial isogeny

Let K be a field and suppose E and E' are isogenous non-isomorphic elliptic curves defined over K. Then there is an isogeny $\pi : E \to E'$ defined over K such that $\ker \pi \cong \mathbb{Z}/n\mathbb{Z}$ for some integer $n > 1$. In particular, the pair $(E, \ker \pi)$ is a non-cuspidal K-rational point of $X_0(n)$. When $n \geq 2$ and $X_0(n)$ has genus 0, we have the Klein-Fricke parameterizations which parameterize the j-invariants of the K-rational points of $X_0(n)$. By using these parameterizations, Cremona, Watkins, and Tsukazaki gave an algorithm to compute the isogeny class of an elliptic curve $E/\mathbb{Q}$. In this talk, we discuss an improvement of this algorithm by means of an explicit classification of isogeny graphs for elliptic curves $E/\mathbb{Q}$ that admit a non-trivial isogeny. We conclude by discussing joint work with Chimarro, Roy, Sahajpal, Tobin, and Wiersema, which uses this explicit classification to investigate how the Kodaira-Néron types of elliptic curves change under 2- or 3-isogeny.