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Solutions of the Ginzburg-Landau equations with vorticity concentrating near a nondegenerate geodesic

It is well-known that under suitable hypotheses, for a sequence of solutions of the (simplified) Ginzburg-Landau equations $-\Delta u_\varepsilon + \varepsilon^{-2}(|u_\varepsilon|^2 - 1)u_\varepsilon = 0$, the energy and vorticity concentrate as $\varepsilon \to 0^+$ around a codimension 2 stationary varifold — a (measure theoretic) minimal surface. Much less is known about the question of whether, given a codimension 2 minimal surface, there exists a sequence of solutions for which the given minimal surface is the limiting concentration set. The corresponding question is very well-understood for minimal hypersurfaces and the scalar Allen-Cahn equation, and for the Ginzburg-Landau equations when the minimal surface is locally area-minimizing, but otherwise quite open.

We consider this question on a 3-dimensional closed Riemannian manifold (M, g), and we prove that any embedded nondegenerate closed geodesic can be realized as the asymptotic energy/vorticity concentration set of a sequence of solutions of the Ginzburg-Landau equations.